TECHNISCHE
UNIVERSITÄT
DRESDEN

Fakultät Mathematik und Naturwissenschaften Institut für Kern- und Teilchenphysik

APPLICATIONS OF MONTE-CARLO METHODS IN PARTICLE PHYSICS

Hauptseminar Monte-Carlo Methods

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How to find the Higgs?

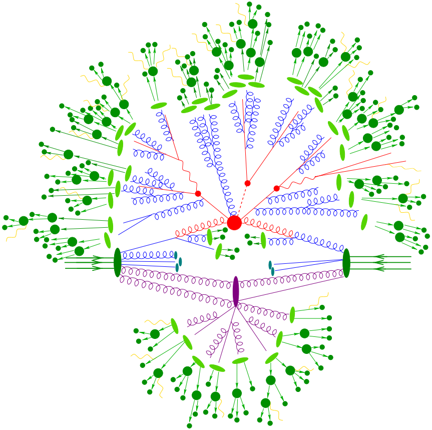
Go to Stockholm in December:

Peter Higgs

1964: “Broken symmetries, massless particles and gauge fields”

2013: Nobel Prize in Physics (with F. Englert)

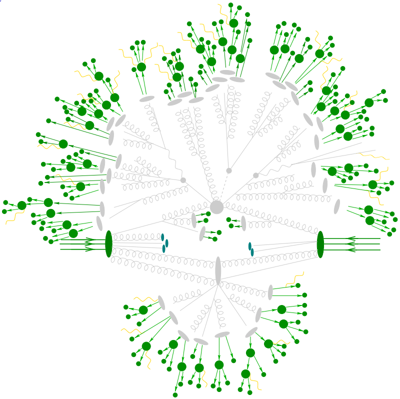
Or look for it in LHC collisions:



Simulation of LHC collisions

- In the detector we can only measure stable particles (mainly hadrons)
- From first principles we can predict the core of such an event
- Simulation programs for all stages are crucial to
  - Extract fundamental theory from measurement
  - Optimise analysis methods on simulated data
  - Understand (& correct for) detector effects

⇒ **Based on Monte-Carlo methods**
i.e. algorithms using (pseudo-)random numbers
→ talk by David



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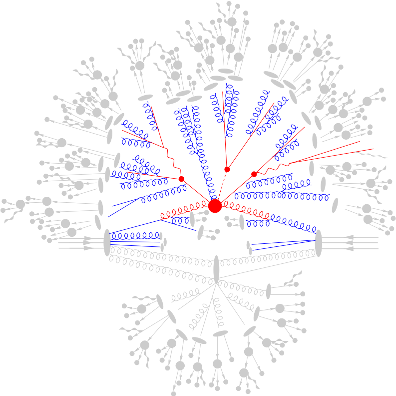

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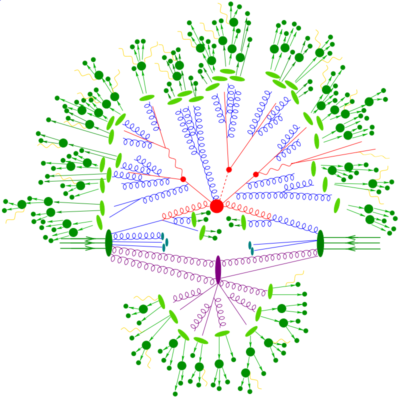


Simulation of the core process

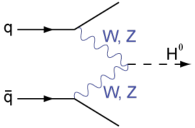
Core process

- Probability distribution in phase space calculable from fundamental theory
- Perturbative series depicted by Feynman diagrams
- Involves fundamental particles only (e.g. e^-, quarks, photons, H-boson, …)

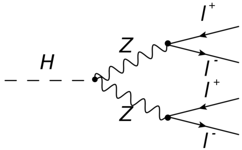
Tasks involving Monte-Carlo methods

- Phase space integration of probability distribution over outgoing momenta to get total probability ($\rightarrow$ number of events at the LHC)
- Sampling phase-space points according to the given probability distribution $\rightarrow$ natural event simulation

Example: Higgs boson production and decay



"Vector-boson fusion" production



Decay to 4 leptons

- Total process: $2 \rightarrow 6$ particles
- Probability distribution in phase space: $\mathcal{M}(p_q, p_{\bar{q}}, p_1, \ldots, p_6)$
- Each particle has 3 (spatial) degrees of freedom; 4-momentum conservation

$\Rightarrow$ Integration in 14 dimensions for total probability of this process

$$\int \mathrm{d}\Phi(p_1, \ldots, p_6)\ \mathcal{M}(p_q, p_{\bar{q}}, p_1, \ldots, p_6)$$

with "Lorentz-invariant phase space element" ($\rightarrow$ talk by Martin)

$$\mathrm{d}\Phi(p_1, \ldots, p_6) = \prod_{i=1}^{6} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i}\ (2\pi)^4\ \delta^4(p_q + p_{\bar{q}} - \sum_{i=1}^{n} p_i)$$

High-dimensional integration

Example from above: Integration in 14 dimensions

$$\int \mathrm{d}\Phi(p_1, \ldots, p_6)\; \mathcal{M}(p_q, p_{\bar{q}}, p_1, \ldots, p_6)$$

$$\mathrm{d}\Phi(p_1, \ldots, p_6) = \prod_{i=1}^{6} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i}\; (2\pi)^4\; \delta^4(p_q + p_q - \sum_{i=1}^{n} p_i)$$

- Analytic integral expressions not generally available
- Complicated integration boundaries and integrands
- Classical integration methods inefficient for high-dimensional integrals (→ talk by Max)

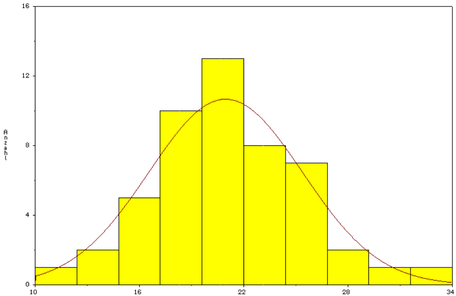
⇒ Method of choice: Monte-Carlo integration (→ talk by Lukas)
with various optimisations (→ talks by Fabian, Christian, Johannes)

Next step for simulation: Generate events

- Integration gave us total number of expected Higgs events
- Now we want to know what they look like – wait, we already know:

$$\mathcal{M}(p_q, p_{\bar{q}}, p_1, \ldots, p_6) \equiv \text{probability distribution}$$

- For studies of the process: Need samples of events with this probability distribution ($\equiv$ "sampling", or "event generation"; $\rightarrow$ talk by Philipp)



Parton shower cascade

Core process done. What now?

- We now have generated $2 \to 6$ scattering events
- Outgoing leptons will hit the detector ✓
- Outgoing quarks are not free according to QCD ✗

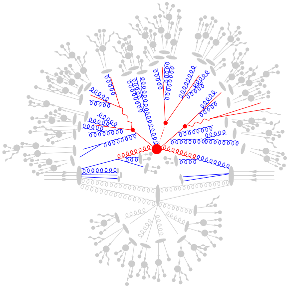
$\Rightarrow$ Further simulation steps necessary

QCD bremsstrahlung

Excursion to your particle physics lecture:

- Partons (= quarks and gluons): charged under the strong interaction (QCD)
- They undergo QCD bremsstrahlung (e.g. gluon emission), like electromagnetically charged electrons undergo EM bremsstrahlung (γ emission)

$\Rightarrow$ Parton cascade



Parton shower Monte-Carlo algorithm

- As we will see (→ talk by Alexander), the parton shower cascade can be simulated by sampling from the following type of probability:

$$\mathcal{P}(t) = f(t) \exp\Big(- \int_0^t f(t') \, \mathrm{d}t' \Big)$$

  for a given $f(t)$.
- There is a surprisingly simple MC algorithm ("veto algorithm") which allows to do that for (almost) arbitrary $f(t)$

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Further simulation steps

Hadronisation, hadron decays, . . .

- Partons after bremsstrahlung cascade will be “slow” enough to form hadrons
- Due to QCD confinement no perturbative first-principles approach possible, instead phenomenological modelling
- Primary hadrons often unstable, simulation of their decay necessary
- MC methods used for sampling of:
  - primary hadron species according to models
  - flavours and momenta of secondary hadrons from decay

Detector simulation

What do our (perfect) particles look like in a (non-perfect) detector?

- Passage of particles through matter:
  simulate energy deposits according to given probability function
- Simulate particle identification with efficiency $\neq 100\%$

Again many applications of MC methods!

Conclusions

Summary

- Different Monte-Carlo methods are used in the context of particle physics
- Simulation of theory predictions and also detector response crucial
- Each stage in such simulations relies on MC methods

Outlook

- In the next weeks we'll learn about the methods in more detail and see an example of them
- Example for today: full event simulation for $pp \to e^+e^-$ at the LHC
  - Watch integration with 310k PS points
  - Look at event structure
  - Sample 500, 5k, and 50k points and fill them into a histogram of $m_{e^+e^-} = \sqrt{[p^\mu(e^+) + p^\mu(e^-)] \cdot [p_\mu(e^+) + p_\mu(e^-)]}$