Sampling generalised radioactive decay type distributions

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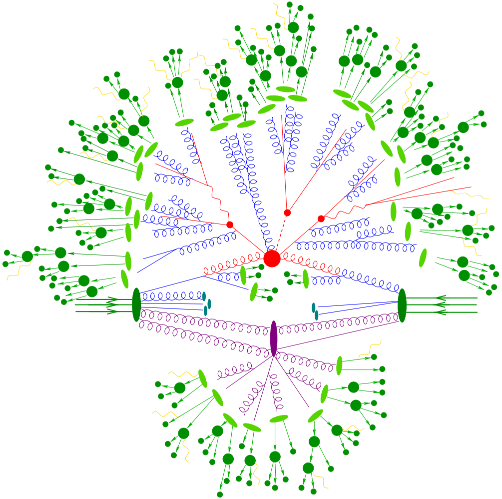
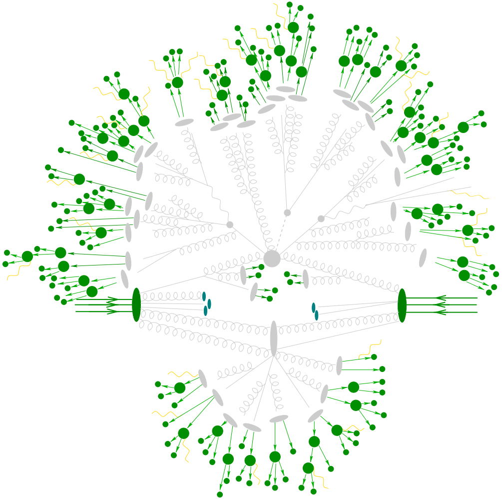
Outline

Core event

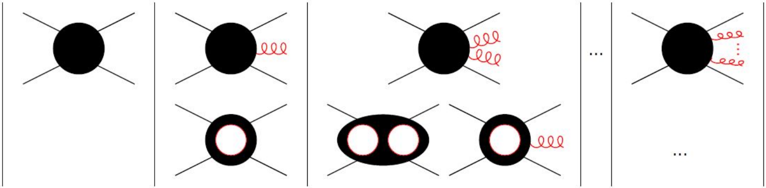


Core event

We can only mesure color neutral Hadrons.



Pertubative series



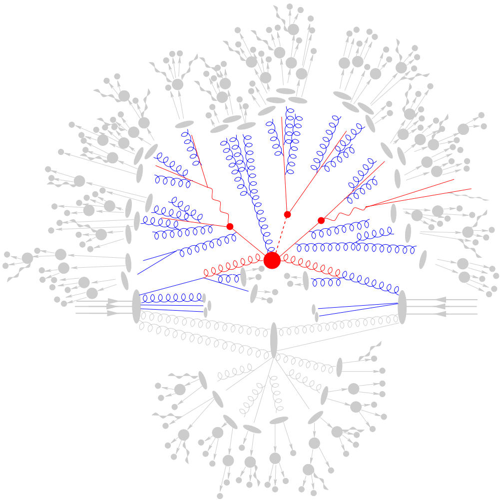
~ 1 $\sim \alpha_s$ $\sim \alpha_s^2$ $\sim \alpha_s^n$

- Exact calculations possible up to $\mathcal{O}(\alpha_s^2)$
- Precision of $\alpha_s^2 \approx 1\%$ is spoiled in certain phase space regions, due to large logarithms $\Rightarrow$ finite remainders of infrared divergences

Core event

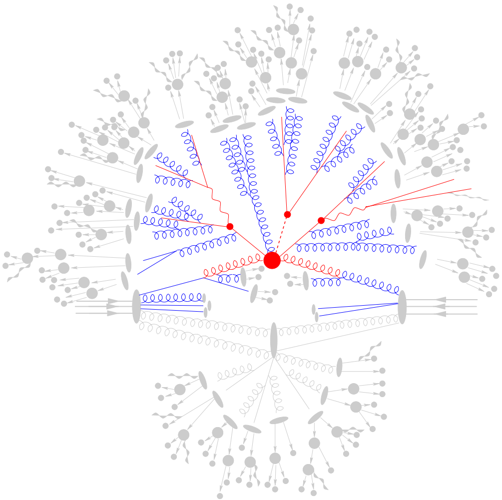
Accuracy of perturbation series is insufficient.

Predictions based on first principals are limited.



Parton shower

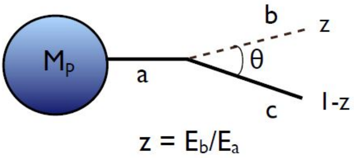
- Prepares hadronization
- Resumes logs to all orders
- No limits on multiplicity
- Computationally cheap



Basics

Matrix elements show divergencies for final state particles, which are close in phase space:

$$\frac{1}{(p_b + p_c)^2} \simeq \frac{1}{2E_b E_c(1-\cos\theta)}$$



factorization for small θ:

$$|M_{p+1}|^2 \simeq |M_p|^2 \frac{1}{t} 8\pi\alpha_s P(z)$$

$$\mathrm{d}\Phi_{p+1} \simeq \mathrm{d}\Phi_p \mathrm{d}t \frac{1}{16\pi^2} \mathrm{d}z \frac{\mathrm{d}\phi}{2\pi}$$

Splitting functions

$$P_{q\to qg}(z) = \frac{4}{3}\cdot\frac{1+z^2}{1-z}$$

$$P_{g\to gg}(z) = 3\cdot\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

$$P_{g\to q\bar{q}}(z) = \frac{n_f}{2}\cdot(z^2+(1-z)^2)$$

$$P_{q\to q\gamma}(z) = e_q^2\cdot\frac{1+z^2}{1-z}$$

$$P_{l\to l\gamma}(z) = e_l^2\cdot\frac{1+z^2}{1-z}$$

Sudakov form faktor

Resumed branching probability in analogy to evolution of ensemble of radioactive nuclei:

Radioactive decay

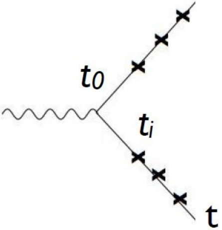
- Constant differntial decay probability
  $f(t) = \lambda$
- Resummed decay probability
  $-\frac{\mathrm{d}\mathcal{N}}{dt} = \lambda \mathcal{N}(t)$
  $\Rightarrow \mathcal{N}(t) \sim \exp(-\lambda t)$
  $\Rightarrow \mathcal{P}(t) = f(t)\mathcal{N}(t)$
  $\sim \lambda \exp(-\lambda t)$

Parton shower branching

- Differntial branching probability
  $f(t) = \frac{1}{t}\frac{\alpha_s}{2\pi}P(z)$
- Resummed branching probability
  $-\frac{\mathrm{d}\mathcal{N}}{dt} = f(t)\mathcal{N}(t)$
  $\Rightarrow \mathcal{N}(t) \sim \exp(-\int_0^t \mathrm{d}t' f(t'))$
  $\Rightarrow \mathcal{P}(t) = f(t)\mathcal{N}(t)$
  $\sim f(t)\exp(-\int_0^t \mathrm{d}t' f(t'))$

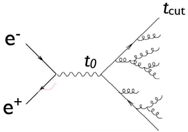
Parton shower recursion

- generate next branching "time" t with
  $\mathcal{P}(t) = f(t)\exp(-\int_{t_0}^{t} \mathrm{d}t' f(t'))$
- solve Sudakov form faktor to obtain t
  $t = F^{-1}[F(t_0) + \ln(R)]$



If $f(t, z)$ is not sufficiently nice ($F(t)$ and/or F^{-1} unknown), this ansatz fails.

1. start with $i = 0$ ant $t_0 = 0$;
2. add 1 to i and select $t_i = G^{-1}(G(t_{i-1}) - \ln(R))$, i.e. according to $g(t)$, but with the constraint that $t_c > t_i > t_{i-1}$;
3. compare a (new) R with the ratio $\frac{f(t_i)}{g(t_i)}$;if $\frac{f(t_i)}{g(t_i)} < R$, then return to point 2 for a new try;
4. otherwise t_i is retained as final answer.



$R \hat{=}$ random number uniform distributed in $[0;1]$
$g(t) \hat{=}$ sufficently nice function with $g(t) > f(t)\ \forall t$
$t_c \hat{=}$ break condition

This works and here is why:

The probability that the first try works is:
$$\mathcal{P}_0(t) = \exp\left(-\int_0^t g(t')\mathrm{d}t'\right) g(t)\frac{f(t)}{g(t)} = f(t)\exp\left(-\int_0^t g(t')\mathrm{d}t'\right)$$

Chances for only one rejection are:
$$\mathcal{P}_1(t) =$$
$$\int_0^t \mathrm{d}t_1 \exp\left(-\int_0^{t_1} g(t_1')\mathrm{d}t_1'\right) g(t_1)\left(1-\frac{f(t_1)}{g(t_1)}\right)\exp\left(-\int_{t_1}^t g(t')\mathrm{d}t'\right) g(t)\frac{f(t)}{g(t)}$$

$$\mathcal{P}_1(t) = \mathcal{P}_0(t)\int_0^t \mathrm{d}t_1 (g(t_1) - f(t_1))$$

To obtain $\mathcal{P}_2$ one has to consider two intermediate times, which yields:

$$\mathcal{P}_2(t) = \mathcal{P}_0(t) \int_0^t \mathrm{d}t_1(g(t_1) - f(t_1)) \int_{t_1}^t \mathrm{d}t_2(g(t_2) - f(t_2))$$
$$= \mathcal{P}_0(t)\frac{1}{2}\left(\int_0^t \mathrm{d}t^{'}(g(t^{'}) - f(t^{'}))\right)^2$$

therefore

$$\mathcal{P}(t) = \sum_{i=0}^{\infty} \mathcal{P}_i(t) = \mathcal{P}_0 \sum_{i=0}^{\infty} \frac{1}{i!}\left(\int_0^t \mathrm{d}t^{'}(g(t^{'}) - f(t^{'}))\right)^i$$
$$= f(t)\exp\left(-\int_0^t g(t^{'})\mathrm{d}t^{'}\right)\exp\left(-\int_0^t \left(g(t^{'}) - f(t^{'})\right)\mathrm{d}t^{'}\right)$$
$$= f(t)\exp\left(-\int_0^t f(t^{'})\mathrm{d}t^{'}\right)$$

Summary

Parton shower can

- Resum logs to all orders
- Compute unlimited particles
- Compute fast
- Prepare hadronization

Parton shower can not

- Compute hard and well separated partons

Outlook

Matrixelement resolves those shortcomings. $\Rightarrow$ merging required!

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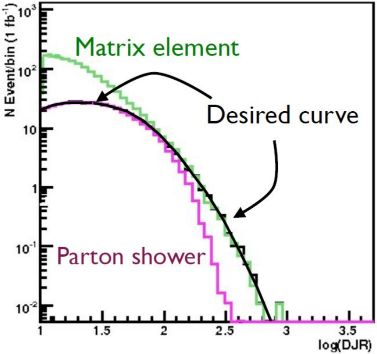
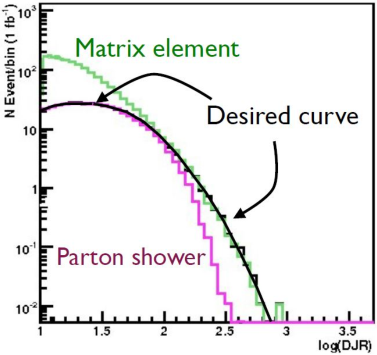


Outlook

Matrixelement resolves those shortcomings. ⇒ merging required!



Different schemes available e.g. MLM or CKKW

Thank you

Sources

- http://arxiv.org/abs/hep-ph/0603175
  Section 4, in particular 4.2
- fsiegert.web.cern.ch/fsiegert/talks/2014-01-16-Dresden.pdf
- https://cp3.irmp.ucl.ac.be/projects/madgraph/attachment/wiki/SchoolNTU/NTU-MLM-lectures.pdf
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