Merging tree matrix elements with truncated showers

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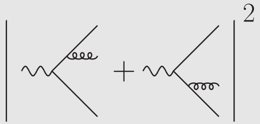
Higher-order tree-level calculations

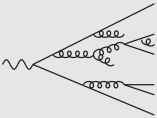
Two approaches

Matrix Elements



- $+$ Exact to fixed order
- $+$ Include all interferences
- $+$ $N_C = 3$ (summed or sampled)
- $-$ Perturbation breaks down due to large logarithms
- $-$ Only low FS multiplicity

Parton Showers



- $+$ Resum logarithmically enhanced contributions to all orders
- $+$ Produce high-multiplicity final state
- $-$ Only approximation to ME for splitting
- $-$ No interference effects
- $-$ Large N_C limit only

$$\Downarrow$$

Goal: Combine advantages

- Describe **particular final state** by **ME** (hard QCD radiation)
- Don't spoil the **inclusive picture** provided by the **PS** (intrajet evolution)

Basis: QCD evolution

Ellis, Stirling, Webber: QCD and Collider Physics

Evolution equation in terms of Sudakov form factor Δ

$$\frac{\partial}{\partial \log(t/\mu^2)} \frac{g_a(z,t)}{\Delta_a(\mu^2,t)} = \frac{1}{\Delta_a(\mu^2,t)} \int_z^{\zeta_{\max}} \frac{\mathrm{d}\zeta}{\zeta} \sum_{b=q,g} \mathcal{K}_{ba}(\zeta,t)\, g_b(z/\zeta,t)$$

$$\Delta_a(\mu^2,t) \;=\; \exp\left\{ -\int_{\mu^2}^{t} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int \mathrm{d}\zeta \sum_{b=q,g} \frac{1}{2} \mathcal{K}_{ab}(\zeta,\bar{t}) \right\}$$

- Kernel describes parton splitting: $\mathcal{K}_{ab}(z,t) \rightarrow \frac{1}{\mathrm{d}\sigma_a^{(N)}(\Phi_N)} \frac{\mathrm{d}\sigma_b^{(N+1)}(z,t;\Phi_N)}{\mathrm{d}\log(t/\mu^2)\,\mathrm{d}z}$
- Solution: Probability for no (forward) shower branching between two scales

$$\mathcal{P}_{\mathrm{no},\,a}(t,t') = \frac{\Delta_a(\mu^2,t')}{\Delta_a(\mu^2,t)} \stackrel{!}{=} \mathcal{R}$$

$\Rightarrow$ **MC method** for dicing successive branching scales using random number $\mathcal{R} \in [0,1]$

Preparation for ME/PS merging

Use different splitting kernels in different regions in phase space, but:
Preserve total evolution equation!

Preparation: Slicing the phase space

Emission phase space divided by parton separation criterion $Q_{ab}(z,t)$

$$\mathcal{K}^{\rm PS}_{ab}(z,t) = \mathcal{K}_{ab}(z,t)\,\Theta\left[Q_{\rm cut} - Q_{ab}(z,t)\right] \quad \text{and} \quad \mathcal{K}^{\rm ME}_{ab}(z,t) = \mathcal{K}_{ab}(z,t)\,\Theta\left[Q_{ab}(z,t) - Q_{\rm cut}\right]$$

- $Q_{ab}(z,t)$ has to identify logarithmically enhanced phase space regions
- Similar to a jet measure

Evolution factorises

- Sudakov form factor:

$$\Delta_a(\mu^2,t) = \Delta^{\rm PS}_a(\mu^2,t')\,\Delta^{\rm ME}_a(\mu^2,t')$$

- No-branching probability:

$$\mathcal{P}_{\text{no},\,a}(t,t') = \mathcal{P}^{\rm PS}_{\text{no},\,a}(t,t')\,\mathcal{P}^{\rm ME}_{\text{no},\,a}(t,t')$$

Simple rules so far for each regime:

- **Independent evolution** according to no-branching probabilities (e.g. by MC-method)
- **Veto** emissions below/above $Q_{\rm cut}$

Want to use exact matrix elements in ME regime

- Seems trivial: Use exact matrix elements as kernel, instead of approximation
- But: Integration in terms of shower variables unfeasible for high multiplicity
- Alternative Idea: Start from ME generated event, where the integration can be optimised

Examples possible with tree ME generator Comix JHEP12(2008)039

- $pp \to 8$ jets
- $pp \to t\bar{t} + 6$ jets
- $pp \to W/Z + 6$ jets
- $pp \to \gamma\gamma + 6$ jets
- $gg \to 12\,g$

Outline of algorithm

1. Generate ME event above $Q_{\rm cut}$ according to σ and $d\sigma$ ✓

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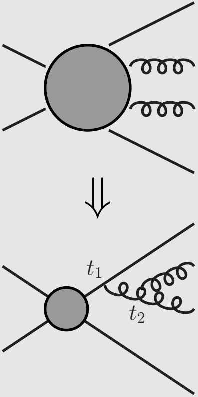
Translate ME event into shower language

Problem: ME only gives final state, no history
Solution: Backward-clustering (running the shower reversed)

1. Take N-particle final state
2. Identify most probable splitting (lowest shower measure)
3. Recombine partons using inverted shower kinematics
   $\rightarrow$ N-1 particles + splitting variables for one node
4. Repeat 2 and 3 until core process

$\Downarrow$

Most probable branching history a la shower.
Now let's use it ...



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3. Reweight $\alpha_s(\mu^2) \rightarrow \alpha_s(p_\perp^2)$ for each branching ✓
4. Start shower evolution for ME regime $\Rightarrow$ **Reject** events containing emission

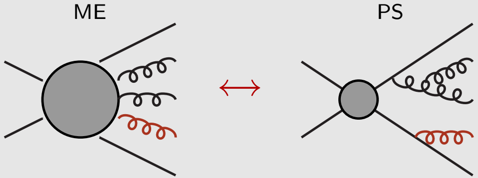
Merging algorithm: Emissions in ME regime

Interpretation of $\mathcal{P}^{\mathrm{ME}}_{\mathrm{no},\,a}(t,t')$

- Vetoed shower **above** Q_{cut}
- Truncated at production and decay scale t', t

Has to be allowed to preserve full QCD evolution.

What if something is emitted?



Emissions in this regime should be described by MEs!

Consequences

- Reduction of cross section $\sigma \to \sigma \cdot \mathcal{P}^{\mathrm{ME}}_{\mathrm{no},\,a}(t,t')$
- Compensated by higher order ME's

$\Rightarrow$ Leading order cross section stable

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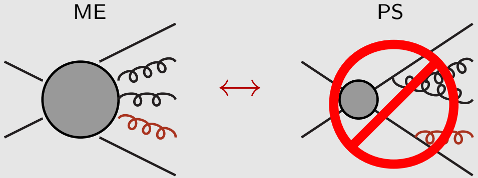


Emissions in this regime should be described by MEs!
$\Rightarrow$ Reject event to avoid double counting

Consequences

- Reduction of cross section $\sigma \to \sigma \cdot \mathcal{P}^{\mathrm{ME}}_{\mathrm{no},\,a}(t,t')$
- Compensated by higher order ME's

$\Rightarrow$ Leading order cross section stable

Outline of algorithm

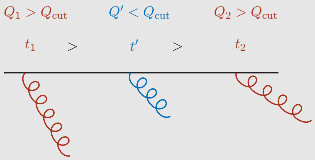
1. Generate ME event above Q_{cut} according to σ and $d\sigma$ ✓
2. Translate ME event into shower language: **Branching history** ✓
3. Reweight $\alpha_s(\mu^2) \rightarrow \alpha_s(p_\perp^2)$ for each branching ✓
4. Start shower evolution for ME regime $\Rightarrow$ **Reject** events containing emission ✓
5. Start shower evolution for PS regime $\Rightarrow$ **Add emissions**

Interpretation of $\mathcal{P}^{\mathrm{PS}}_{\mathrm{no},\,a}(t,t')$

- Vetoed shower **below** Q_{cut}
- **Truncated** at production and decay scale t', t

Truncated shower

Some splittings are pre-determined by ME



Mismatch of Q and t allows intermediate radiation!
$\Rightarrow$ "Truncated" shower necessary to fill phase space below Q_{cut}

1. Q_{cut}-vetoed shower between t_1 and t_2
2. Then insert pre-determined node t_2
3. Restart evolution from there

Outline of algorithm

1. Generate ME event above Q_{cut} according to σ and $d\sigma$ ✓
2. Translate ME event into shower language: **Branching history** ✓
3. Reweight $\alpha_s(\mu^2) \to \alpha_s(p_\perp^2)$ for each branching ✓
4. Start shower evolution for ME regime $\Rightarrow$ **Reject** events containing emission ✓
5. Start shower evolution for PS regime $\Rightarrow$ **Add emissions** ✓

$$\Downarrow$$

Evolution according to $\mathcal{P}_{\text{no},\,a}(t,t') = \mathcal{P}^{\text{PS}}_{\text{no},\,a}(t,t')\,\mathcal{P}^{\text{ME}}_{\text{no},\,a}(t,t')$ **preserved**
Emissions above Q_{cut} **ME-corrected**

Is it relevant? Results for $p\bar{p} \to e^+e^- + \text{jets}$ at $\sqrt{s} = 1960\,\text{GeV}$

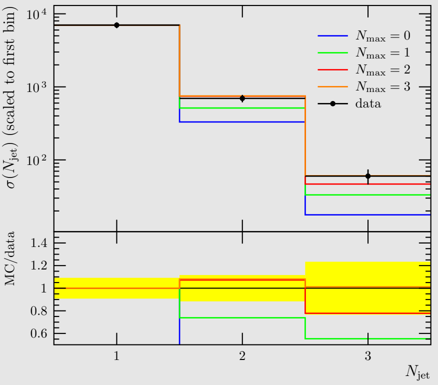
PRL 100,102001 arXiv:0711.3717 [hep-ex]

Algorithm implemented in SHERPA framework

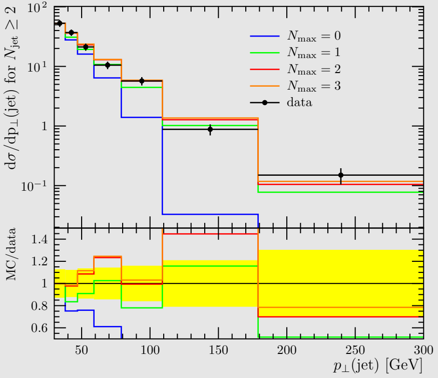
CSSHOWER++ Shower based on Catani-Seymour subtraction

COMIX Matrix elements based on Berends-Giele recursion

Jet multiplicity



$p_\perp$(jet) in $N_{\text{jet}} \geq 2$ events

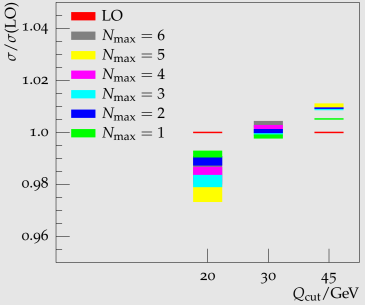


Is it consistent? Results for $p\bar{p} \to e^+e^- + \text{jets}$ at $\sqrt{s} = 1960\,\text{GeV}$

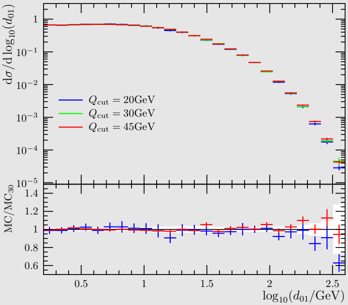
Consistency tests

- Total LO cross section stable?
- Observables independent from "unphysical" merging cut?

Total cross sections



$1 \to 0$ jet resolution ($k_\perp$)

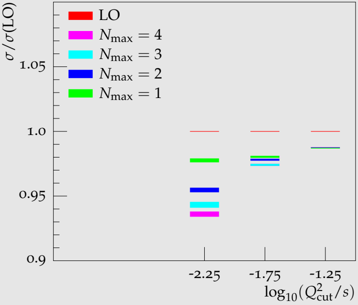


Is it consistent? Results for $e^+e^- \to$ jets at $\sqrt{s} = 91\,\mathrm{GeV}$

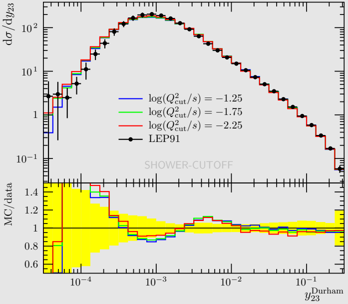
Consistency tests

- Total LO cross section stable?
- Observables independent from "unphysical" merging cut?

Total cross sections



$3 \to 2$ jet resolution in Durham measure



Backup

Parton separation criterion

Reminder

$$\mathcal{K}_{ab}^{\rm PS}(z,t) = \mathcal{K}_{ab}(z,t)\,\Theta\left[Q_{\rm cut} - Q_{ab}(z,t)\right] \quad \text{and} \quad \mathcal{K}_{ab}^{\rm ME}(z,t) = \mathcal{K}_{ab}(z,t)\,\Theta\left[Q_{ab}(z,t) - Q_{\rm cut}\right]$$

- $Q_{\rm cut}$ has to regularise QCD radiation MEs (like a jet resolution)
- Otherwise completely arbitrary until now

$$Q_{ij}^2 \;=\; 2\,p_i p_j\; \min_{k\neq i,j} \frac{2}{C_{i,j}^k + C_{j,i}^k}$$

Final state partons $(ij) \to i,\, j$

$$C_{i,j}^k \;=\; \begin{cases} \dfrac{p_i p_k}{(p_i+p_k)p_j} - \dfrac{m_i^2}{2\,p_i p_j} & \text{if } j=g \\ 1 & \text{else} \end{cases}$$

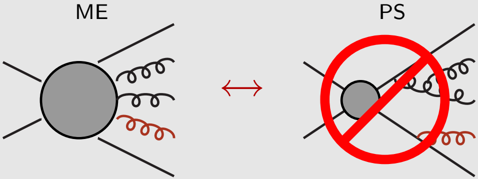
Initial state parton $a \to (aj)\,j$

$$C_{a,j}^k \;=\; C_{(aj),\,j}^k$$

with $p_{aj} = p_a - p_j$

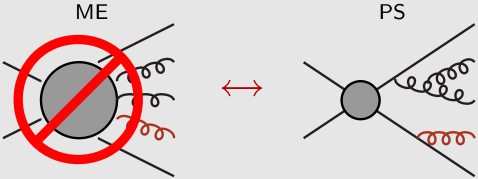
- The minimum is over all possible colour partners k of parton (ij)
- Identifies regions of soft ($E_g \to 0$) and/or (quasi-)collinear ($\approx k_\perp^2 \to 0$) enhancements
- Similar to jet resolution (e.g. Durham in e^+e^- case), but with flavour information

- So far: Rejection of emissions in ME regime ⇒ Sudakov weighted MEs



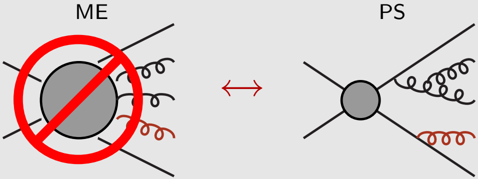
Highest multiplicity treatment

- So far: Rejection of emissions in ME regime ⇒ Sudakov weighted MEs



What if higher order ME not available?

- So far: Rejection of emissions in ME regime ⇒ Sudakov weighted MEs



What if higher order ME not available?

Highest multiplicity events

- $N = N_{\max}$ emissions from ME ⇒ correct branching probability up to scale of last ME emission, $t_{\min}$ (global, for all legs)
- PS must account for all emissions $t < t_{\min}$, even if $Q > Q_{\text{cut}}$
- Implemented by employing standard PS evolution beyond last ME emission

⇓

Hard radiation respected
Remaining phase space filled