NLO matrix elements and parton showers

Seminar Heidelberg, 4 Nov 2010

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Based on

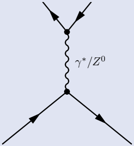
- arXiv:1009.1127 (Stefan Höche, Frank Krauss, Marek Schönherr, FS)
- arXiv:1008.5399 (Stefan Höche, Frank Krauss, Marek Schönherr, FS)
- arXiv:0912.3501 (Stefan Höche, Steffen Schumann, FS)
- arXiv:0903.1219 (Stefan Höche, Frank Krauss, Steffen Schumann, FS)

LHC phenomenology

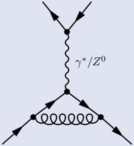
- Higgs/BSM signals with heavy particles decaying into high multiplicity final states
- Backgrounds from simple SM processes with many additional jets

⇒ Need good understanding of higher order QCD corrections to SM processes

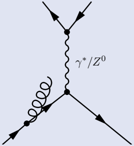
Typical framework: Calculation to fixed order in α_s, e.g. NLO



Born level matrix element



Virtual matrix element



Real emission matrix element

This talk

Improving approximate resummation of this series with exact fixed order corrections

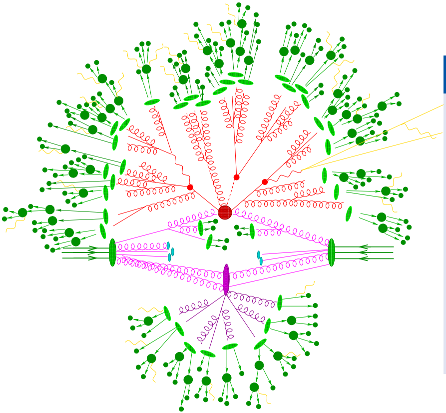
Table of Contents

Monte-Carlo event generation

What are event generators?

- Simulation programs for collider physics
- Modelling of the complete hadronic final state

⇒ Work horses for theoretical interpretation of measurements



Basic principle

- Factorisation into event phases
- Perturbatively calculable:
  - Hard scattering
  - Initial state parton shower
  - Final state parton shower
  - (Multiple parton interactions)
- Non-perturbative modelling:
  - (Multiple parton interactions)
  - Hadronisation
  - Hadron decays

Central ingredient: Parton showers

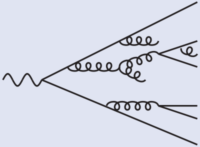
Motivation

- Higher-order QCD corrections to hard scattering:
  Infrared divergences from real/virtual cancel for inclusive quantities (→ KLN)
- But: Resolution through confinement of partons at $\mu_{\rm had} \approx 1$ GeV (hadronisation)
  $\Rightarrow$ Not inclusive
- Finite remainders of infrared singularities:
  logarithms of ratio $\mu_F^2/\mu_{\rm had}^2$ with each $\mathcal{O}(\alpha_s)$
- Such large logarithms have to be resummed to all orders

Parton shower:

- Higher orders represented by parton branchings

$\Rightarrow$ Evolution of parton ensemble between μ_F^2 and $\mu_{\rm had}^2$



Question

How to get the (no-)branching probabilities to describe this evolution between different scales?

Construction of a parton shower (I/II)

Factorisation of QCD emissions

Universal factorisation of QCD real emission ME in soft/collinear limit:

$$\mathcal{R} \;\to\; \mathcal{B} \times \left(\sum_{ij,k \in \text{partons}} \frac{1}{2p_i p_j}\, 8\pi\alpha_s\, \mathcal{K}_{ij,k}(p_i,p_j,p_k) \right)$$

- $\mathcal{B}$ Born matrix element
- Sum over subterms ij, k of the factorisation, e.g. parton lines (DGLAP) potentially with spectator k
- $\frac{1}{2p_i p_j}$ from massless propagator
  Evolution variable of shower $t \sim 2p_i p_j$ (e.g. $k_\perp$, angle, ...)
- $\mathcal{K}_{ij,k}$ splitting kernel for branching $(ij) + k \to i + j + k$
  Specific form depends on scheme of the factorisation above, e.g.:
  - Altarelli-Parisi splitting functions
  - Dipole terms from Catani-Seymour subtraction
  - Antenna functions
  - ...

Construction of a parton shower (II/II)

Differential (no-)branching probability

- Radiative phase space: $\mathrm{d}\Phi_{\mathrm{rad}}^{ij,k} = \frac{1}{16\pi^2}\,\mathrm{d}t\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}$
- Combined with radiative part of the factorised ME (Jacobian/symmetry factor/PDFs ignored)

$$\mathrm{d}\sigma_{\mathrm{rad}}^{ij,k} = \frac{\mathrm{d}t}{t}\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,\frac{\alpha_s}{2\pi}\,\mathcal{K}_{ij,k}$$

Differential branching probability

No-branching probability

Above: Differential probability for one branching to (not) happen in interval $\mathrm{d}t$
Goal: Total no-branching probability between scale t' and t''

- Integrate over z, ϕ, and t from t' to t''
- Assume multiple independent emissions (Poisson statistics) ⇒ Exponentiation

subterm: $$\Delta_{ij,k}(t',t'') = \exp\left\{-\sum_{f_i=q,g}\int_{t'}^{t''}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\mathrm{d}z\int_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\,\frac{\alpha_s}{2\pi}\,\mathcal{K}_{ij,k}(z,t)\right\}$$

event: $$\Delta(t',t'') = \prod_{ij,k}\Delta_{ij,k}(t',t'')$$

Master formula

Cross section up to first emission in a parton shower

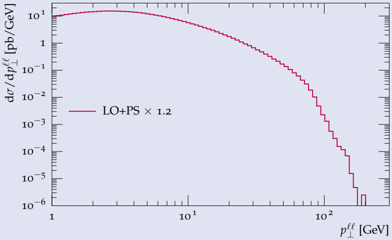
$$\sigma = \int \mathrm{d}\Phi_B \, \mathrm{B} \left[\underbrace{\Delta(t_0, \mu^2)}_{\text{unresolved}} + \underbrace{\sum_{ij,k} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_-}^{z_+} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} \, \Delta(t, \mu^2) \frac{8\pi\,\alpha_s}{2p_i p_j} \, \mathcal{K}_{ij,k}(z,t)}_{\text{resolved}} \right]$$

Features

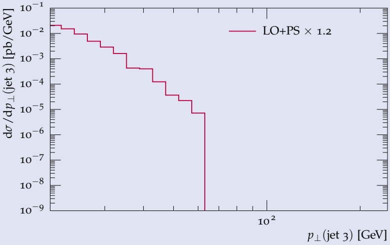
- LO weight B for Born-like event
- Unitarity: Term in square brackets $\left[\ldots\right] = 1 \Rightarrow$ LO cross section preserved
- “Unresolved” part: No emissions above parton shower cutoff t_0
- “Resolved” part: Emission between t_0 and factorisation scale μ^2
- Emission in parton shower approximation with $\mathcal{K}_{ij,k}$

Canonical Example: Drell-Yan process $pp \to \ell\ell$

Transverse momentum of the lepton pair



Transverse momentum of the third jet



Conclusions

- $p_\perp^{\ell\ell}$ probes QCD emissions because of recoil
- Resummation avoids divergence of fixed order calculation for $p_\perp^{\ell\ell} \to 0$
- Hard QCD emissions (leading to $p_\perp^{\ell\ell} > \mu_F^2 \approx m_Z$) not well described (as we will see later)
- Factor $K = 1.2$ to compare to NLO results later

ME+PS formalism

Main idea of ME+PS merging

Phase space slicing for QCD radiation in shower evolution

- Hard emissions $Q_{ij,k}(z,t) > Q_{\text{cut}}$
  - Events rejected
  - Compensated by events starting from higher-order ME (regularised by Q_{cut})

  $\Rightarrow$ Splitting kernels replaced by exact real emission matrix elements

$$\frac{8\pi\alpha_s}{2p_ip_j}\mathcal{K}_{ij,k}(z,t) \quad\rightarrow\quad \frac{8\pi\alpha_s}{2p_ip_j}\,\mathcal{K}^{\text{ME}}_{ij,k}(z,t) \;=\; \frac{\mathcal{R}_{ij,k}}{\mathcal{B}}$$

- Soft/collinear emissions $Q_{ij,k}(z,t) < Q_{\text{cut}}$

  $\Rightarrow$ Retained from parton shower $\qquad \mathcal{K}_{ij,k}(z,t) = \mathcal{K}^{\text{PS}}_{ij,k}(z,t)$

Note

- Boundary determined by “jet criterion” $Q_{ij,k}$
  - Has to identify soft/collinear divergences in MEs, like jet algorithm
  - Otherwise arbitrary, but some choices better than others
- In both regions: No-branching probabilities still from shower

$$\Delta(t',t'') \quad\rightarrow\quad \Delta^{(\text{PS})}(t',t'')$$

Master formula

Cross section up to first emission in ME+PS

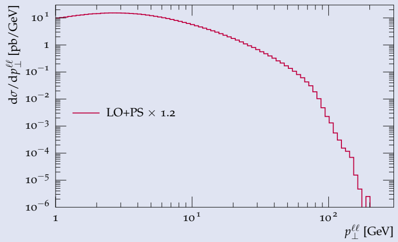
$$\sigma = \int \mathrm{d}\Phi_B \,\mathrm{B} \left[\underbrace{\Delta^{(\mathrm{PS})}(t_0,\mu^2)}_{\text{unresolved}} + \sum_{ij,k} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_-}^{z_+} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} \, \Delta^{(\mathrm{PS})}(t,\mu^2) \right.$$
$$\left. \times \left(\underbrace{\frac{8\pi\,\alpha_s}{2p_ip_j}\, \mathcal{K}^{(\mathrm{PS})}_{ij,k}(z,t)\, \Theta(Q_{\mathrm{cut}} - Q_{ij,k})}_{\text{resolved, PS domain}} + \underbrace{\frac{\mathrm{R}_{ij,k}}{\mathrm{B}}\, \Theta(Q_{ij,k} - Q_{\mathrm{cut}})}_{\text{resolved, ME domain}} \right) \right]$$

Features

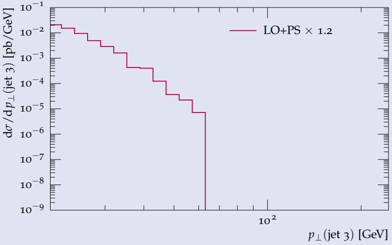
- LO weight B for Born-like event
- Unitarity slightly violated due to mismatch of $\Delta^{(\mathrm{PS})}$ and R/B
  $[\ldots] \approx 1 \Rightarrow$ LO cross section only approximately preserved
- Unresolved emissions as in parton shower approach
- Resolved emissions now sliced into PS and ME domain
- Only for one emission here, but possible up to very high number of emissions

Canonical Example: Drell-Yan process $pp \to \ell\ell$

Transverse momentum of the lepton pair



Transverse momentum of the third jet



Canonical Example: Drell-Yan process $pp \to \ell\ell$

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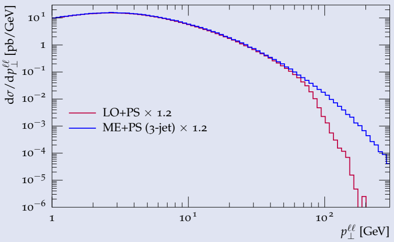
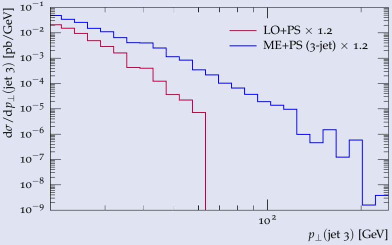


Transverse momentum of the third jet



Conclusions

- Multiple hard emissions properly accounted for
- Resummation preserved
- Inclusive rate still at LO $\Rightarrow$ factor $K = 1.2$ necessary

Results: Features and shortcomings

Example

Diphoton production at Tevatron

- Recently published by DØ Phys.Lett.B690:108-117,2010
- Isolated hard photons with:
  - $E_\perp^{\gamma^1} > 21$ GeV
  - $E_\perp^{\gamma^2} > 20$ GeV
  - $|\eta_\gamma| < 0.9$
  - Isolation: $E_\perp(R = 0.4) - E_\perp^\gamma < 2.5$ GeV
- Here: Azimuthal angle between the diphoton pair

ME+PS simulation using SHERPA 1.2.2 with QCD+QED interleaved shower and merging

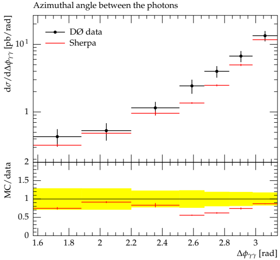
Höche, Schumann, FS (2010)

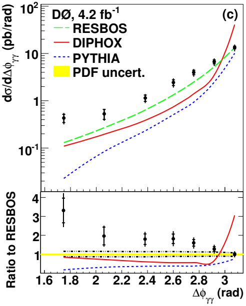
Conclusions

Shapes described very well even for this non-trivial process/observable for both:

- Hard region, e.g. $\Delta\Phi_{\gamma\gamma} \to 0$
- Soft region, e.g. $\Delta\Phi_{\gamma\gamma} \to \pi$

Total cross section too low ⇒ Virtual MEs needed





The POWHEG method

Motivation

- Parton shower for resummation ✓
- ME+PS for correct hard radiation pattern ✓
- Inclusive rate still at LO in α_s . . . can we do NLO + parton shower?
  - MC@NLO Frixione, Webber (2002)
  - POWHEG Nason (2004), Frixione, Nason, Oleari (2007) (used in the following)

Two issues to solve

1. Cross section at NLO accuracy in α_s
2. Radiation pattern of first emission according to real ME

Note

- Completely orthogonal to and independent of ME+PS merging
- Only possible for first emission, not for higher orders

Cross section at NLO accuracy in α_s

Reminder: Matrix elements contributing to NLO

- Born ME → automatic tree-level generators
- Virtual ME → dedicated codes, Binoth Les Houches interface
- Real emission ME → automatic tree-level generators

Integrating over real emission phase space

- Problem: Cancellation of infrared divergences between virtual and real, Separate numerical integration (N and $N+1$ final states) not possible
- Solution: Subtraction procedure, e.g. Catani-Seymour or Frixione-Kunszt-Signer
  - Subtract universal divergent terms from real ME (S)
  - Integrate them analytically and add to virtual ME (I)
  - ⇒ Poles cancel
- Integration of real emission phase space explicitely or by Monte-Carlo sampling

⇒ NLO weight for event with Born level kinematics

$$\overline{\mathrm{B}} = \mathrm{B} + \mathrm{V} + \mathrm{I} + \sum_{\{\widetilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \int \mathrm{d}\Phi_{R|B}^{ij,k} \left[\mathrm{R}_{ij,k} - \mathrm{S}_{ij,k} \right]$$

Radiation pattern of first emission

Matrix element corrections in parton showers

- Well-known method for reinstating $\mathcal{O}(\alpha_s)$ accuracy in parton shower radiation pattern
- Feasible only for simple cases

Reweighting principle (simplified)

- From above: weight with which to correct one emission

$$w_{ij,k} = \frac{\mathrm{d}\sigma^{ij,k}_{\mathrm{rad}}}{\mathrm{d}\sigma^{(\mathrm{PS})\,ij,k}_{\mathrm{rad}}} = \frac{2\,p_i p_j}{8\pi\,\alpha_s}\,\frac{\mathcal{R}_{ij,k}}{\mathcal{B}\,\mathcal{K}_{ij,k}}\;.$$

- Determine overestimate W_{ij} for the total weight throughout real-emission phase space
- Replace splitting kernels in parton shower $\mathcal{K}_{ij,k} \to W_{ij}\mathcal{K}_{ij,k}$
- Accept shower branchings only with probability w/W

Master formula

Cross section up to first emission in POWHEG

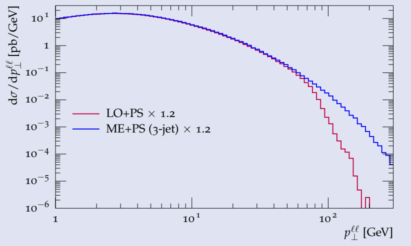
$$\sigma = \int \mathrm{d}\Phi_B\, \overline{\mathrm{B}} \left[\underbrace{\Delta^{(\mathrm{ME})}(t_0,\mu^2)}_{\text{unresolved}} + \underbrace{\sum_{ij,k} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_-}^{z_+} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, \Delta^{(\mathrm{ME})}(t,\mu^2) \frac{\mathrm{R}_{ij,k}}{\mathrm{B}}}_{\text{resolved}} \right]$$

Features

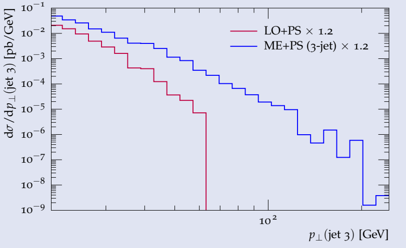
- NLO weight $\overline{\mathrm{B}}$ for Born-like event
- Unitarity: Term in square brackets $\left[\ldots\right] = 1$
  $\Rightarrow$ NLO cross section preserved
- First resolved emission exact according to real emission ME
- No-branching probability $\Delta^{(\mathrm{ME})}(t_0,\mu^2)$ from R/B instead of $\mathcal{K}$
- Only one corrected emission, further emissions in parton shower approximation

Canonical Example: Drell-Yan process $pp \to \ell\ell$

Transverse momentum of the lepton pair



Transverse momentum of the third jet



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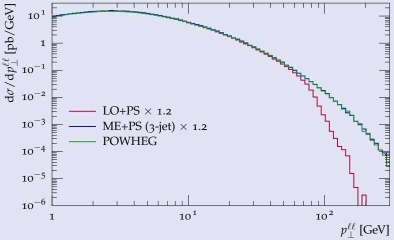

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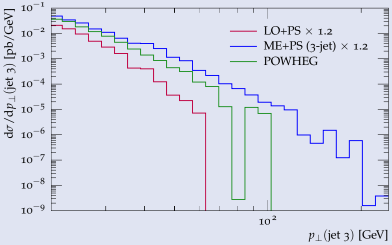


Conclusions

- Inclusive rate at NLO $\Rightarrow$ no K-factor necessary
- First hard emission properly accounted for
  $\Rightarrow$ Observables sensitive to first emission (e.g. $p_\perp^{\ell\ell}$) fine
- Further emissions only in parton shower approximation
  $\Rightarrow$ Observables sensitive to higher order corrections not sufficiently described

The MENLOPS algorithm

Motivation

Two different methods to improve parton showers:

- POWHEG
  - + NLO accuracy in cross section
  - + First emission according to real emission ME
  - + Soft/collinear resummation from parton shower
  - – Further hard emissions in parton shower approximation
- ME+PS
  - – Only LO accuracy in cross section
  - + Soft/collinear resummation from parton shower
  - + All hard emissions according to real emission ME

Can we combine both methods and get rid of their disadvantages?

Idea starting from ME+PS

(see also Hamilton, Nason (2010))

- Replace “unresolved” and “PS resolved” part in ME+PS with POWHEG
  i.e. run POWHEG generator instead of normal parton shower for first emission
- Generate “resolved ME” part separately through real emission MEs as before
- Supply real ME events with local K-factor $\frac{\overline{\mathrm{B}}}{\mathrm{B}}$
  formally beyond NLO, but necessary for smooth merging

Master formula

Cross section up to first emission in MENLOPS

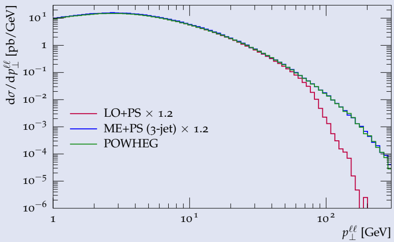
$$\sigma = \int \mathrm{d}\Phi_B \, \overline{\mathrm{B}} \Bigg[\underbrace{\Delta^{(\mathrm{ME})}(t_0, \mu^2)}_{\text{unresolved}} + \sum_{ij,k} \frac{1}{16\pi^2} \int_{t_0}^{\mu^2} \mathrm{d}t \int_{z_-}^{z_+} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} \frac{\mathrm{R}_{ij,k}}{\mathrm{B}} \times \Bigg(\underbrace{\Delta^{(\mathrm{ME})}(t, \mu^2)\, \Theta(Q_{\mathrm{cut}} - Q_{ij,k})}_{\text{resolved, PS domain}} + \underbrace{\Delta^{(\mathrm{PS})}(t, \mu^2)\, \Theta(Q_{ij,k} - Q_{\mathrm{cut}})}_{\text{resolved, ME domain}} \Bigg) \Bigg]$$

Features

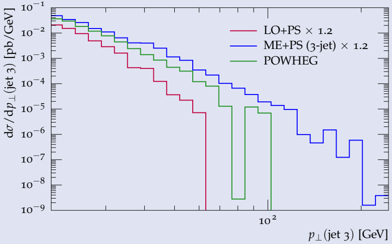
- NLO weight $\overline{\mathrm{B}}$ for Born-like event
- Unitarity still slightly violated: $\big[\ldots\big] \approx 1$
  $\Rightarrow$ NLO cross section only approximately preserved
- R events generated separately (not through POWHEG)
  $\Rightarrow$ has to be supplemented with local $\frac{\overline{\mathrm{B}}(\Phi_B)}{\mathrm{B}(\Phi_B)}$ explicitely to reproduce the above

Canonical Example: Drell-Yan process $pp \to \ell\ell$

Transverse momentum of the lepton pair



Transverse momentum of the third jet



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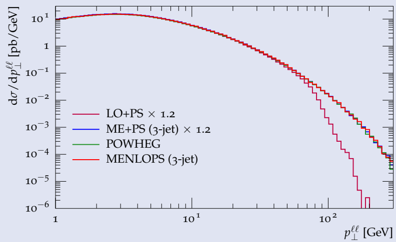

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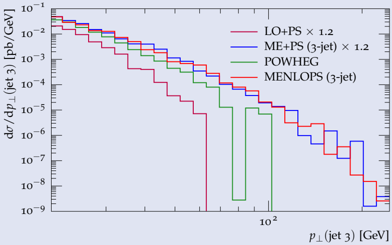
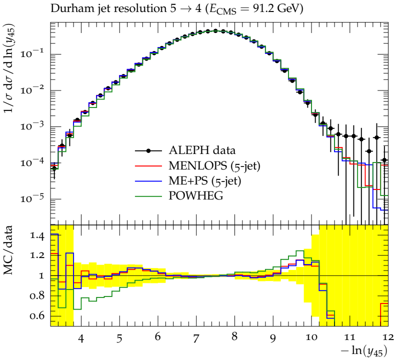


Conclusions

Jack-of-all-trades algorithm

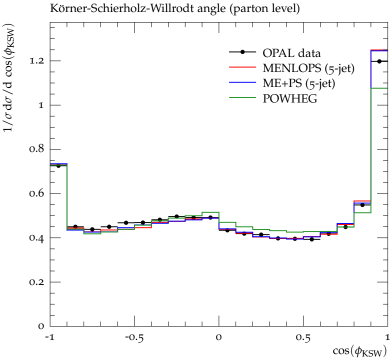
- Inclusive rate at NLO $\Rightarrow$ no K-factor necessary
- Multiple hard emissions properly accounted for

Comparison to LEP results for $e^+e^- \to$ hadrons



Jet resolution where 5 jets are clustered into 4 jets

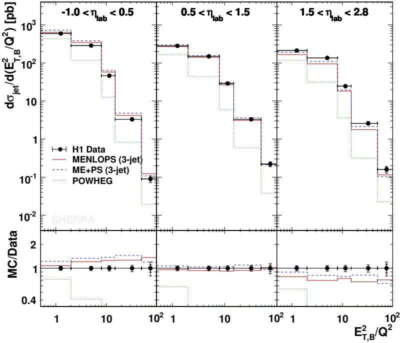
Eur. Phys. J. C35 (2004), 457-486



KSW Angle built from momenta of four most energetic jets

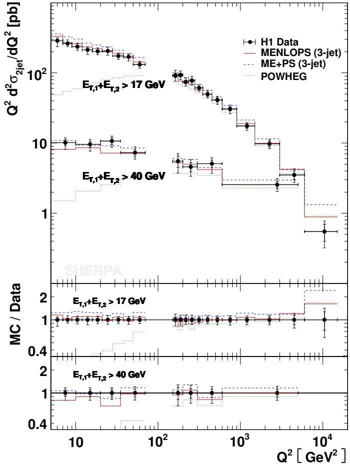
arXiv:hep-ex/0101044

Comparison to HERA results for Deep-Inelastic lepton-nucleon Scattering



Inclusive jet cross section as function of transverse energy in Breit frame

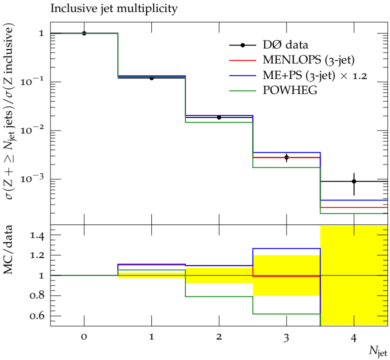
arXiv:hep-ex/0206029



Dijet cross section as function of Q^2

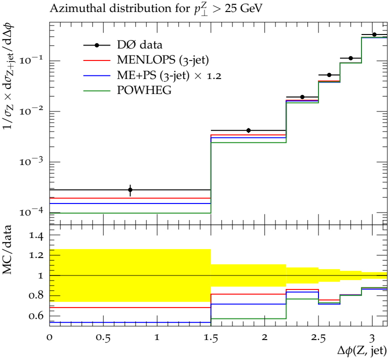
arXiv:hep-ex/0010054

Comparison to Tevatron results for $pp \to \ell\ell$



Inclusive jet multiplicity

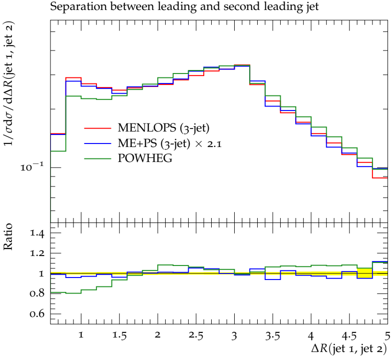
arXiv:hep-ex/0608052



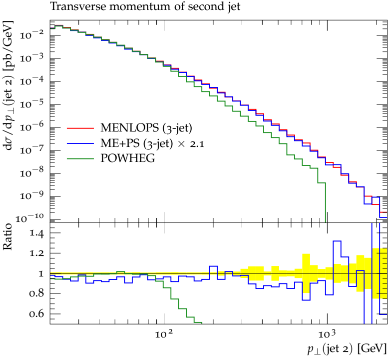
Azimuthal separation of lepton pair and leading jet

arXiv:0907.4286

Predictions for Higgs-production via gluon fusion at LHC

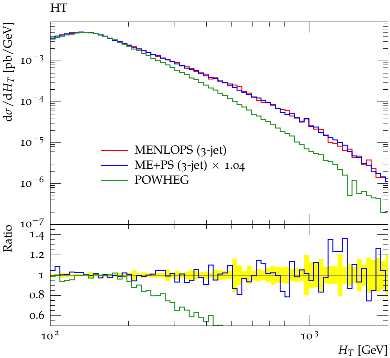


Separation between leading and second leading jet

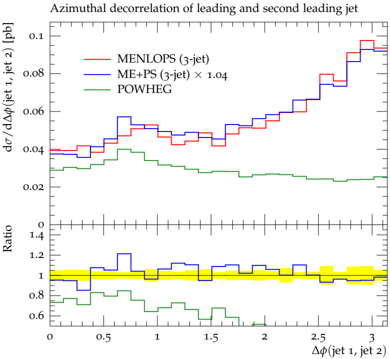


Transverse momentum of second leading jet

Predictions for W^+W^- production at LHC



Scalar sum of missing E_T and transverse momenta of jets and leptons



Azimuthal decorrelation between leading and second leading jet

Conclusions and outlook

Conclusions

- Tree-level ME+PS merging works well for shapes, but needs K-factor for cross section
- POWHEG reproduces full NLO cross section and shape of first emission but fails for additional hard radiation
- Combination of full NLO and higher order tree-level MEs with shower achieves both of the above
- Recently much progress and already first implementations
- Automation within SHERPA framework
- Full NLO only in core process, not in higher order corrections . . .

Outlook

- . . . yet
- Application to more processes
- Public availability in a SHERPA release,
  as simple to use as tree-level merging