Hard scattering in MC event generators

CERN ATLAS Team Physics workshop
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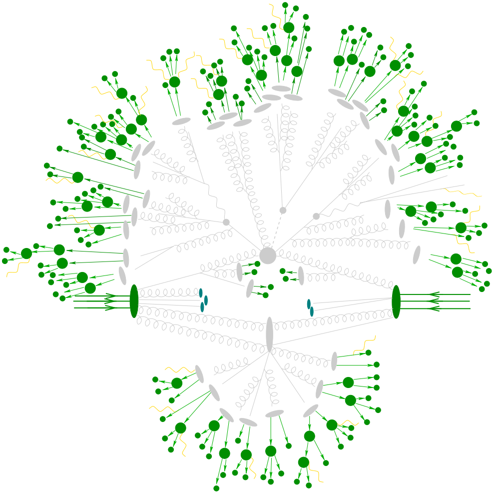
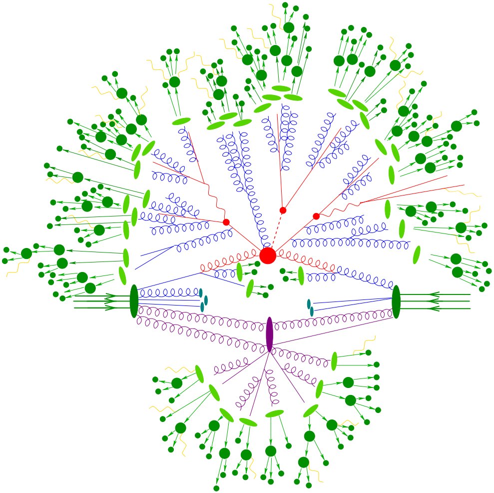
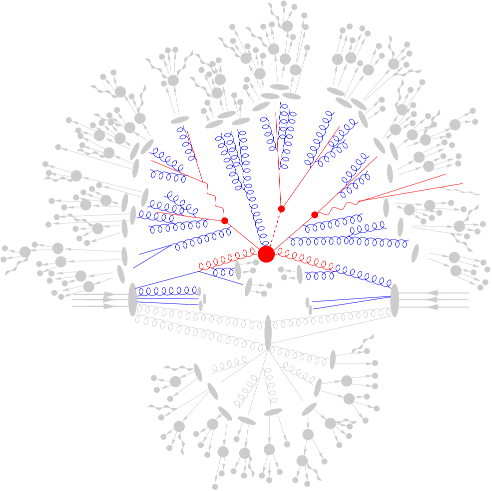


- We want:
  Simulation of $pp \to$ full hadronised final state



- We want:
  Simulation of $pp \to$ full hadronised final state
- MC event representation for $pp \to t\bar{t}H$



- We want:
  Simulation of $pp \to$ full hadronised final state
- MC event representation for $pp \to t\bar{t}H$
- We know from first principles:
  - Hard scattering at fixed order in perturbation theory (**Matrix Element**)
  - Approximate resummation of QCD corrections to all orders (**Parton Shower**)
- Missing bits:
  Hadronisation/Underlying event $\to$ Peter's talk

- We know from first principles:
  - Hard scattering at fixed order in perturbation theory (**Matrix Element**)
  - Approximate resummation of QCD corrections to all orders
    (**Parton Shower**)

Outline

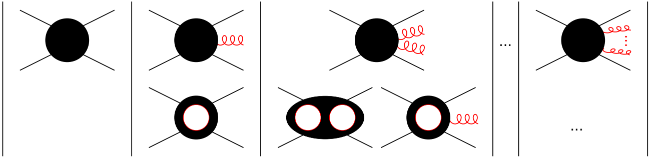
- Reminder: Perturbation theory
- Fixed-order calculations for QCD corrections
- The parton shower approximation to QCD corrections
- Combining the two above
  - Tree-level ME+PS
  - NLO+PS
  - (Combining the two above)

Not covered

- Electro-weak corrections
- BFKL-like simulation

- We know from first principles:
  - Hard scattering at fixed order in perturbation theory (**Matrix Element**)
  - Approximate resummation of QCD corrections to all orders (**Parton Shower**)

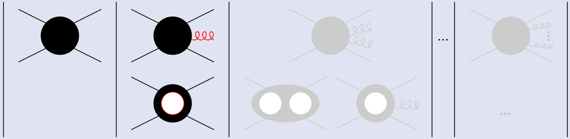
- Too stupid to solve QCD and calculate e.g. $pp \to t\bar{t}H$ exactly
- But can calculate parts of the perturbative series in α_s:



~ 1 $\sim \alpha_s$ $\sim \alpha_s^2$

- Exact calculations possible up to $\mathcal{O}(\alpha_s^2)$ for some processes
- All orders known (and resummed) only in approximation
- $\exists$ advantages/disadvantages in both cases

Components



$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_B \left[\mathcal{B}(\Phi_B) + \mathcal{V}(\Phi_B) + \mathcal{I}^{(\mathrm{S})}(\Phi_B)\right] + \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R) - \mathcal{D}^{(\mathrm{S})}(\Phi_R)\right]$$

$\mathcal{B}$orn level/$\mathcal{R}$eal emission
: Automated tree-level calculators available for a long time

Subtraction procedure ($\mathcal{D}$, $\mathcal{S}$)
: Automated implementations available for a few years

$\mathcal{V}$irtual matrix elements
: Loop amplitudes starting to become automated only recently

Note

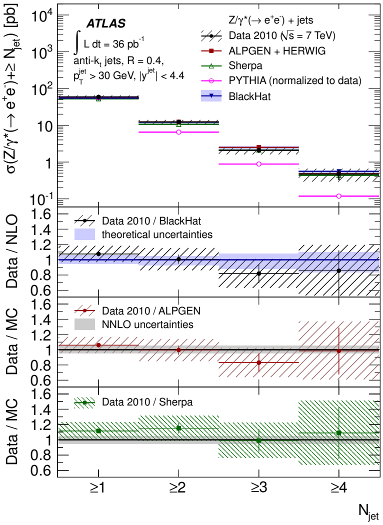
Analytical resummation of enhanced logarithmic terms to all orders available for some distributions (e.g. ResBos, HqT, Caesar). No event generator though.

Features

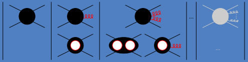
- $+$ NLO accurate cross section
- $+$ Reduced uncertainties
- $+$ Jets gain structure (jet $\neq$ parton)
- $-$ Non-perturbative effects not included

Status

- Process specific calculations available for $2 \to 2, 3, 4$ processes on the Les-Houches wishlist
- Many processes in MCFM
- State-of-the-art example: $W/Z + 4$ jets with BlackHat+Sherpa



- ATLAS data arXiv:1111.2690
- BlackHat+Sherpa arXiv:1108.2229



Features

- NNLO accuracy and further reduction in scale uncertainties
- Important if NLO corrections are large and for benchmark processes
- Subtraction procedure much more involved $\Rightarrow$
  Only inclusive cross section results for a long time

Recently: Examples of fully exclusive NNLO calculations

- $gg \to H$: HNNLO [Catani, Grazzini], FEHiP [Anastasiou, Melnikov, Petriello]
- $pp \to W/Z$: FEWZ [Melnikov, Petriello], DYNNLO [Catani, Cieri, de Florian, Ferrera, Grazzini]
- $e^+e^- \to$ 3 jets [Gehrmann, Gehrmann, Glover, Heinrich; Weinzierl]
- $H \to b\bar{b}$ decay [Anastasiou, Lazopoulos, Herzog]
- $pp \to WH$ [Ferrera, Tramontano, Grazzini]
- $pp \to \gamma\gamma$ [Catani, Cieri, de Florian, Ferrera, Grazzini]

$\Rightarrow$ Fiducial cuts can be applied!

Fixed order calculations not sufficient to describe soft/collinear partons, e.g.:

- $p_\perp^Z \to 0$
- QCD Bremsstrahlung before hadronisation

What happens?

- Soft/collinear emission is $\sim \alpha_s \Rightarrow$ higher orders should be suppressed
- But: Soft/collinear emission comes with large (logarithmic) enhancement factor

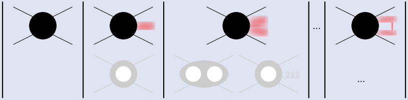
$\Rightarrow$ Perturbation series does not converge

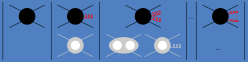
Solution

Approximation of real emission matrix element $\mathcal{R}$ from Born $\mathcal{B}$:

$$\mathcal{R} \xrightarrow{ij\text{ collinear}} \mathcal{B} \times \left(\sum_{\{ij\}} \frac{1}{2p_i p_j} 8\pi\alpha_s \, \mathcal{K}_{ij}(p_i, p_j) \right)$$

- Emissions described by parton shower kernels $\mathcal{K}$ (e.g. Altarelli-Parisi)
- Factorisation into core and emission
  $\Rightarrow$ Can be repeated for all orders





Main idea of "ME+PS merging" a la CKKW-L

[Catani, Krauss, Kuhn, Webber (2001); Lonnblad (2001); Höche, Krauss, Schumann, FS (2009)]

Phase space slicing for QCD radiation in shower evolution

- Soft/collinear emissions $Q_{ij} < Q_{\rm cut}$
  $\Rightarrow$ Retained from parton shower approximation $\mathcal{K}_{ij}$
- Hard emissions $Q_{ij} > Q_{\rm cut}$
  - Events rejected
  - Compensated by adding events with higher-order tree-level ME (above $Q_{\rm cut}$)

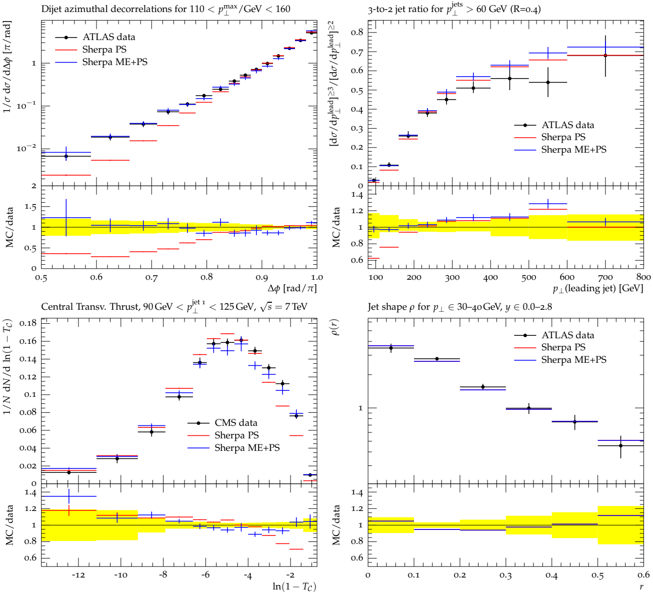
  $\Rightarrow$ Splitting kernels replaced by exact real emission matrix elements

$$\mathrm{B} \times \sum_{\{ij\}} \frac{8\pi\alpha_s}{2p_ip_j}\mathcal{K}_{ij} \;\longrightarrow\; \mathrm{R}$$

Note

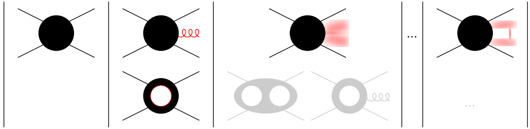
- Boundary determined by "jet criterion" $Q_{ij,k}$
  - Has to identify soft/collinear divergences in MEs, like jet algorithm
  - Otherwise arbitrary, but some choices better than others
- Resummation features from parton shower retained

Example: ME+PS for QCD multi-jet production



- ATLAS $\Delta\phi$
  arXiv:1102.2696
- ATLAS R_{32}
  arXiv:1107.2092
- CMS event shapes
  arXiv:1102.0068
- ATLAS jet shapes
  arXiv:1101.0070

- NLO accuracy needs full calculation including virtuals, but:
  NLO calculations miss non-perturbative effects
- Can we somehow connect them to a parton shower + hadronisation?



Naive Idea

- Each term in NLO calculation represents separate event sample:

$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_B \left[\mathcal{B}(\Phi_B) + \mathcal{V}(\Phi_B) + \mathcal{I}^{(\mathrm{S})}(\Phi_B)\right] + \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R) - \mathcal{D}^{(\mathrm{S})}(\Phi_R)\right]$$

- Apply PS resummation to 5 samples separately

Does it work? No: [Frixione, Webber (2002)]
If $\mathcal{R}$ and $\mathcal{D}$ are showered separately $\Rightarrow$ "double counting"

[Frixione, Webber (2002)]

$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_B \left[\mathcal{B}(\Phi_B) + \mathcal{V}(\Phi_B) + \mathcal{I}^{(\mathrm{S})}(\Phi_B)\right] + \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R) - \mathcal{D}^{(\mathrm{S})}(\Phi_R)\right]$$

MC@NLO construction

- Use parton shower splitting functions instead of $\mathcal{D}$
- Correct for that in the remaining terms
- Apply one-step parton shower to Born-like events

Features

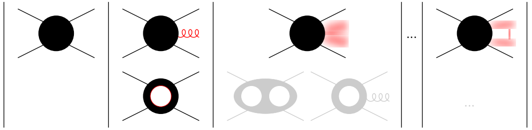
\+ Reproduces $\sigma^{(\mathrm{NLO})}$ to NLO accuracy

\+ Further PS/hadronisation trivially added

- Terms beyond NLO from resummation

(-) Events with negative weights can appear

− Further emissions only in PS approximation



[Nason (2004); Nason, Frixione, Oleari (2007)]

$$\sigma^{(\mathrm{NLO})} = \int \mathrm{d}\Phi_B \left[\mathcal{B}(\Phi_B) + \mathcal{V}(\Phi_B) + \mathcal{I}^{(\mathrm{S})}(\Phi_B)\right] + \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R) - \mathcal{D}^{(\mathrm{S})}(\Phi_R)\right]$$

POWHEG construction

- Choose "$\mathcal{D} = \mathcal{R}$"
  $\Rightarrow$ second term vanishes
- Correct for that in the remaining term by exponentiating $\mathcal{R}$ in a one-step parton shower

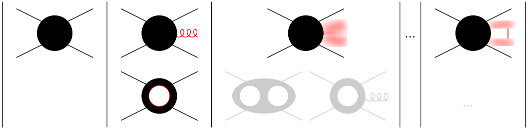
Features

+ Reproduces $\sigma^{(\mathrm{NLO})}$ to NLO accuracy
+ Further PS/hadronisation trivially added

(+) (Almost) no events with negative weights

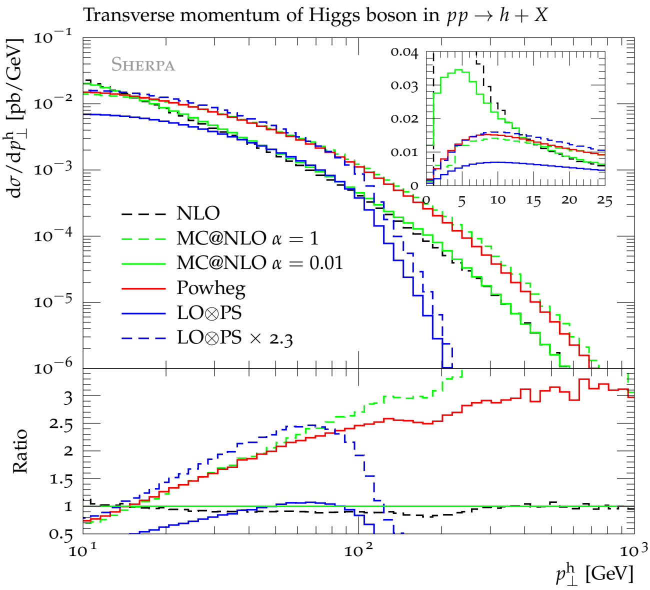
– Uncontrolled/tunable terms beyond NLO from $\mathcal{R}$-exponentiation

– Further emissions only in PS approximation



SHERPA implementation of NLO+PS methods in the example of $gg \to$ Higgs

[Höche, Krauss, Schönherr, FS (2011)]



Yet another approach? Why?

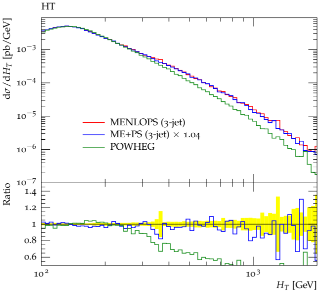
- NLO+PS: predictions for $>$ 1-jet in PS approximation only
- We already know how to restore LO accuracy in PS evolution
- Can this be combined with NLO+PS?

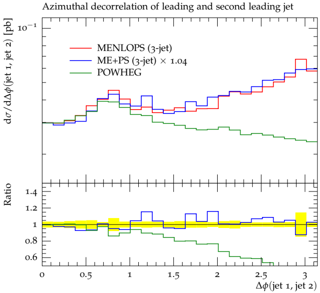
MENLOPS

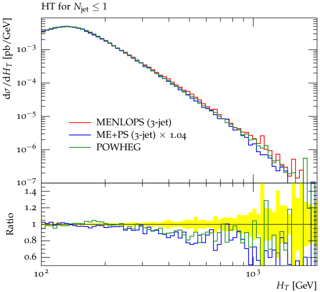
[Hamilton, Nason; Höche, Krauss, Schönherr, FS (2010)]

- Phase space slicing a la ME+PS on top of NLO+PS
- NLO accuracy in core process, LO accuracy for first n jets (typically $n \simeq 5$ feasible)
- In SHERPA publically available since version 1.2.3 using built-in POWHEG

Influence of MENLOPS on observables







Example: W^+W^- production at 14 TeV

- Scalar transverse momenta sum H_T
- Azimuthal separation of the two hardest jets $\Delta\phi$
- H_T after veto of $\geq$ 2-jet events

Summary

- Traditional approaches for QCD corrections: N(N)LO calculation or parton shower
- Progress in recent years ⇒ combination to improve parton showers with fixed-order results
- Tree-level ME+PS for LO accuracy in higher jet multiplicities
- POWHEG/MC@NLO for NLO accuracy in core process
- Combination of both: MENLOPS

Outlook

- One obvious missing feature:
  Merging of e.g. $W + 0, 1, 2, 3, 4$-jet matrix elements at NLO accuracy in each
- Forecast: Will be available in at least 2 independent implementations in 2012

Translate ME event into shower language

Why?

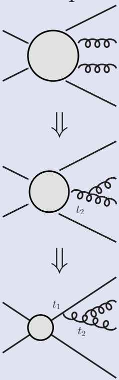
- Need starting scales t for PS evolution
- Have to embed existing emissions into PS evolution

Problem: ME only gives final state, no history

Solution: Backward-clustering (running the shower reversed), similar to jet algorithm:

1. Select last splitting according to shower probablities
2. Recombine partons using inverted shower kinematics $\rightarrow$ N-1 particles + splitting variables for one node
3. Reweight $\alpha_s(\mu^2) \rightarrow \alpha_s(p_\perp^2)$
4. Repeat 1 - 3 until core process ($2 \rightarrow 2$)

Example:



Truncated shower

- Shower each (external and intermediate!) line between determined scales
- "Boundary" scales: factorisation scale μ_F^2 and shower cut-off t_o

Problem

- At NLO, can PS resummation simply be done separately for $\mathcal{B}, \mathcal{V}+\mathcal{I}, \mathcal{R}-\mathcal{D}$?

$$\langle O\rangle^{(\mathrm{NLO})} = \sum_{\vec{f}_\mathrm{B}} \int \mathrm{d}\Phi_B \left[\mathcal{B}(\Phi_B) + \tilde{\mathcal{V}}(\Phi_B) + \sum_{\widetilde{ij}} \mathcal{I}^{(\mathrm{S})}_{\widetilde{ij}}(\Phi_B) \right] O(\Phi_B)$$
$$+ \sum_{\vec{f}_\mathrm{R}} \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R)\, O(\Phi_R) - \sum_{\{ij\}} \mathcal{D}^{(\mathrm{S})}_{ij}(\Phi_R)\, O(b_{ij}(\Phi_R)) \right]$$

- Different observable dependence in $\mathcal{R}$ and $\mathcal{D}$ but if showered separately $\Rightarrow$ "double counting"

Solution: Let's in the following …

- rewrite $\langle O\rangle^{(\mathrm{NLO})}$ a bit
- add some PS resummation into the game leading to $\langle O\rangle^{(\mathrm{NLO+PS})}$ and claim that:
  - $\langle O\rangle^{(\mathrm{NLO+PS})} = \langle O\rangle^{(\mathrm{NLO})}$ to $\mathcal{O}(\alpha_s)$
  - $\langle O\rangle^{(\mathrm{NLO+PS})}$ contains the first step of a PS evolution which can then be continued trivially with a regular PS
- sketch how $\langle O\rangle^{(\mathrm{NLO+PS})}$ is being generated in MC@NLO and POWHEG

First rewrite: Additional set of subtraction terms $\mathcal{D}^{(\mathrm{A})}$

$$\langle O\rangle^{(\mathrm{NLO})} = \sum_{\vec{f}_B}\int \mathrm{d}\Phi_B\, \bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B)\, O(\Phi_B) + \sum_{\vec{f}_R}\int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R)\, O(\Phi_R) - \sum_{\{ij\}} \mathcal{D}^{(\mathrm{A})}_{ij}(\Phi_R)\, O\left(b_{ij}(\Phi_R)\right)\right]$$

with $\bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B)$ defined as:

$$\bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B) = \mathcal{B}(\Phi_B) + \tilde{\mathcal{V}}(\Phi_B) + \sum_{\{\widetilde{\imath\jmath}\}} \mathcal{I}^{(\mathrm{S})}_{\widetilde{\imath\jmath}}(\Phi_B) + \sum_{\{\widetilde{\imath\jmath}\}}\sum_{f_i=q,g}\int \mathrm{d}\Phi^{ij}_{R|B}\left[\mathcal{D}^{(\mathrm{A})}_{ij}(r_{\widetilde{\imath\jmath}}(\Phi_B)) - \mathcal{D}^{(\mathrm{S})}_{ij}(r_{\widetilde{\imath\jmath}}(\Phi_B))\right]$$

- $\mathcal{D}^{(\mathrm{A})}_{ij}$ must have same kinematics mapping as $\mathcal{D}^{(\mathrm{S})}_{ij}$
- Exact choice of $\mathcal{D}^{(\mathrm{A})}_{ij}$ will later specify MC@NLO vs. POWHEG
- Issue with different observable kinematics not yet solved → next step

Second rewrite: Make observable correction term explicit

$$
\begin{aligned}
\langle O\rangle^{(\mathrm{NLO})} &= \sum_{\vec{f}_B} \int \mathrm{d}\Phi_B\, \bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B)\, O(\Phi_B) \\
&\quad + \sum_{\vec{f}_R} \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R) - \sum_{\{ij\}} \mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R) \right] O(\Phi_R) \\
&\quad + \langle O\rangle^{(\mathrm{corr})}
\end{aligned}
$$

with $\langle O\rangle^{(\mathrm{corr})}$ defined as:

$$
\langle O\rangle^{(\mathrm{corr})} = \sum_{\vec{f}_R} \int \mathrm{d}\Phi_R \sum_{\{ij\}} \mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R) \left[O(\Phi_R) - O(b_{ij}(\Phi_R)) \right]
$$

- Explicit correction term due to observable kinematics: $\langle O\rangle^{(\mathrm{corr})}$
- Essence of NLO+PS
  - Ignore $\langle O\rangle^{(\mathrm{corr})}$ for the time being
  - Apply PS resummation to first line using $\Delta^{(\mathrm{A})}$ in which $\mathcal{D}^{(\mathrm{PS})} \to \mathcal{D}^{(\mathrm{A})}$

Master formula for NLO+PS up to first emission

$$\langle O\rangle^{(\mathrm{NLO+PS})} = \sum_{\vec{f}_B}\int \mathrm{d}\Phi_B\,\bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B)\Bigg[\underbrace{\Delta^{(\mathrm{A})}(t_0)}_{\text{unresolved}}\,O(\Phi_B)$$

$$+\sum_{\{\widetilde{ij}\}}\sum_{f_i}\int_{t_0}\mathrm{d}\Phi_{R|B}^{ij}\,\underbrace{\frac{\mathcal{D}_{ij}^{(\mathrm{A})}(r_{\widetilde{ij}}(\Phi_B))}{\mathcal{B}(\Phi_B)}\,\Delta^{(\mathrm{A})}(t)}_{\text{resolved, singular}}\,O(r_{\widetilde{ij}}(\Phi_B))\Bigg]$$

$$+\sum_{\vec{f}_R}\int \mathrm{d}\Phi_R\,\underbrace{\Bigg[\mathcal{R}(\Phi_R)-\sum_{ij}\mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R)\Bigg]}_{\text{resolved, non-singular}}\,O(\Phi_R)$$

- This is generated in the following way:
  - Generate seed event according to first or second line of $\langle O\rangle^{(\mathrm{NLO})}$ on last slide
  - Second line: $\mathbb{H}$-event with Φ_R is kept as-is $\rightarrow$ resolved, non-singular term
  - First line: $\mathbb{S}$-event with Φ_B is processed through one-step PS with $\Delta^{(\mathrm{A})}$ $\Rightarrow$ emission (resolved, singular) or no emission (unresolved) above t_0
- To $\mathcal{O}(\alpha_s)$ this reproduces $\langle O\rangle^{(\mathrm{NLO})}$ including the correction term
- Resolved cases: Subsequent emissions can be generated by ordinary PS