W+n-jet predictions at NLO matched with a parton shower

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Based on

- arXiv:1111.1220 (Stefan Höche, Frank Krauss, Marek Schönherr, FS)
- arXiv:1201.5882 (Stefan Höche, Frank Krauss, Marek Schönherr, FS)

Two approaches to higher-order corrections

Fixed order ME calculation

- + Exact to fixed order
- + Includes all interferences
- + $N_C = 3$ (summed or sampled)
- + Includes virtual contributions
- – Perturbation breaks down in logarithmically enhanced regions
- – Only low FS multiplicity

Parton Shower

- + Resums logarithmically enhanced contributions to all orders
- + High-multiplicity final state
- + Allows for exclusive hadron-level events
- – Only approximation for emission ME
- – Large N_C limit

$\Downarrow$

Goal: Combine advantages

- Include virtual contributions and first hard emission from **NLO ME**
- Add further parton evolution with the **PS**

Factorisation of collinear QCD emissions

- Universal factorisation of QCD real emission ME in collinear limit:

$$\mathcal{R} \overset{ij \text{ collinear}}{\longrightarrow} \mathcal{D}_{ij}^{(\mathrm{PS})} = \mathcal{B} \times \left(\frac{1}{2 p_i p_j} \; 8\pi\alpha_s \; \mathcal{K}_{ij}(p_i, p_j) \right)$$

- Differential branching probability: $\mathrm{d}\sigma_{\mathrm{branch}}^{\widetilde{ij}} = \sum_{f_i=q,g} \mathrm{d}\Phi_{R|B}^{ij}(t,z,\varphi) \, \frac{\mathcal{D}_{ij}^{(\mathrm{PS})}}{\mathcal{B}}$
- Assume multiple independent emissions (Poisson statistics) $\Rightarrow$ Exponentiation yields total no-branching probability down to evolution scale t:

$$\begin{aligned}\Delta^{(\mathrm{PS})}(t) &= \prod_{\widetilde{ij}} \left[1 - \int \mathrm{d}\sigma_{\mathrm{branch}}^{\widetilde{ij}} \, \Theta\left(t(\Phi_{R|B}^{ij}) - t \right) + \ldots \right] \\ &= \prod_{\widetilde{ij}} \exp\left\{ - \sum_{f_i=q,g} \int \mathrm{d}\Phi_{R|B}^{ij} \, \Theta\left(t(\Phi_{R|B}^{ij}) - t \right) \frac{\mathcal{D}_{ij}^{(\mathrm{PS})}}{\mathcal{B}} \right\}\end{aligned}$$

Expectation value of observable $\mathcal{O}$ up to first emission

$$\langle O \rangle^{(\mathrm{PS})} = \int \mathrm{d}\Phi_B \, \mathcal{B} \Bigg[\underbrace{\Delta^{(\mathrm{PS})}(t_0) \, O(\Phi_B)}_{\text{unresolved}} + \underbrace{\sum_{\widetilde{ij}} \sum_{f_i} \int_{t_0}^{\mu_F^2} \mathrm{d}\Phi_{R|B}^{ij} \, \frac{\mathcal{D}_{ij}^{(\mathrm{PS})}}{\mathcal{B}} \, \Delta^{(\mathrm{PS})}(t) \, O\left(r_{\widetilde{ij}}(\Phi_B) \right)}_{\text{resolved}} \Bigg]$$

Reminder + Notation: Subtraction method

- Contributions to NLO cross section: $\mathcal{B}$orn, $\mathcal{V}$irtual and $\mathcal{R}$eal emission
- $\mathcal{V}$ and $\mathcal{R}$ divergent in separate phase space integrations
  $\Rightarrow$ Subtraction method for expectation value of observable O at NLO:

$$\langle O\rangle^{(\mathrm{NLO})} = \sum_{\vec{f}_{\mathrm{B}}} \int \mathrm{d}\Phi_B \left[\mathcal{B}(\Phi_B) + \tilde{\mathcal{V}}(\Phi_B) + \sum_{\widetilde{ij}} \mathcal{I}^{(\mathrm{S})}_{\widetilde{ij}}(\Phi_B) \right] O(\Phi_B)$$

$$+ \sum_{\vec{f}_{\mathrm{R}}} \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R)\, O(\Phi_R) - \sum_{\{ij\}} \mathcal{D}^{(\mathrm{S})}_{ij}(\Phi_R)\, O(b_{ij}(\Phi_R)) \right]$$

- Subtraction terms $\mathcal{D}$ and their integrated form $\mathcal{I}$
  e.g. Frixione, Kunszt, Signer (1995); Catani, Seymour (1996)
- Subtraction defines phase space mappings $\Phi_R \underset{r_{\widetilde{ij}}}{\overset{b_{ij}}{\rightleftharpoons}} \left(\Phi_B, \Phi^{ij}_{R|B}\right)$

Problem

- Applying PS resummation to LO event is simple ✓
- Can the same simply be done separately for $\mathcal{B}$ and $\mathcal{V}+\mathcal{I}$ and $\mathcal{R}-\mathcal{D}$ at NLO?

$$\langle O\rangle^{(\mathrm{NLO})} = \sum_{\vec{f}_\mathrm{B}} \int \mathrm{d}\Phi_B \left[\mathcal{B}(\Phi_B) + \tilde{\mathcal{V}}(\Phi_B) + \sum_{\widetilde{ij}} \mathcal{I}^{(\mathrm{S})}_{\widetilde{ij}}(\Phi_B) \right] O(\Phi_B)$$

$$+ \sum_{\vec{f}_\mathrm{R}} \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R)\, O(\Phi_R) - \sum_{\{ij\}} \mathcal{D}^{(\mathrm{S})}_{ij}(\Phi_R)\, O(b_{ij}(\Phi_R)) \right]$$

- Different observable dependence in $\mathcal{R}$ and $\mathcal{D}$ but if showered separately ⇒ "double counting" ✗

Solution: Let's in the following …

Frixione, Webber (2002)

- rewrite $\langle O\rangle^{(\mathrm{NLO})}$ a bit
- add PS resummation into the game leading to $\langle O\rangle^{(\mathrm{NLO+PS})}$ and claim that:
  - $\langle O\rangle^{(\mathrm{NLO+PS})} = \langle O\rangle^{(\mathrm{NLO})}$ to $\mathcal{O}(\alpha_s)$
  - $\langle O\rangle^{(\mathrm{NLO+PS})}$ contains the first step of a PS evolution which can then be continued trivially with a regular PS
- sketch how $\langle O\rangle^{(\mathrm{NLO+PS})}$ is being generated in MC@NLO formalism

First rewrite: Additional set of subtraction terms $\mathcal{D}^{(\mathrm{A})}$

$$
\begin{aligned}
\langle O\rangle^{(\mathrm{NLO})} &= \sum_{\vec{f}_B} \int \mathrm{d}\Phi_B\, \bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B)\, O(\Phi_B) \\
&\quad + \sum_{\vec{f}_R} \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R)\, O(\Phi_R) - \sum_{\{ij\}} \mathcal{D}^{(\mathrm{A})}_{ij}(\Phi_R)\, O\left(b_{ij}(\Phi_R)\right)\right]
\end{aligned}
$$

with $\bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B)$ defined as:

$$
\begin{aligned}
\bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B) &= \mathcal{B}(\Phi_B) + \tilde{\mathcal{V}}(\Phi_B) + \sum_{\{\widetilde{ij}\}} \mathcal{I}^{(\mathrm{S})}_{\widetilde{ij}}(\Phi_B) \\
&\quad + \sum_{\{\widetilde{ij}\}} \sum_{f_i=q,g} \int \mathrm{d}\Phi^{ij}_{R|B} \left[\mathcal{D}^{(\mathrm{A})}_{ij}(r_{\widetilde{ij}}(\Phi_B)) - \mathcal{D}^{(\mathrm{S})}_{ij}(r_{\widetilde{ij}}(\Phi_B))\right]
\end{aligned}
$$

- $\mathcal{D}^{(\mathrm{A})}_{ij}$ must have same kinematics mapping as $\mathcal{D}^{(\mathrm{S})}_{ij}$
- Exact choice of $\mathcal{D}^{(\mathrm{A})}_{ij}$ will specify e.g. MC@NLO vs. POWHEG
- Issue with different observable kinematics not yet solved → next step

Second rewrite: Make observable correction term explicit

$$\langle O \rangle^{(\mathrm{NLO})} = \sum_{\vec{f}_B} \int \mathrm{d}\Phi_B \, \bar{\mathcal{B}}^{(\mathrm{A})}(\Phi_B) \, O(\Phi_B) + \sum_{\vec{f}_R} \int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R) - \sum_{\{ij\}} \mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R) \right] O(\Phi_R) + \langle O \rangle^{(\mathrm{corr})}$$

with $\langle O \rangle^{(\mathrm{corr})}$ defined as:

$$\langle O \rangle^{(\mathrm{corr})} = \sum_{\vec{f}_R} \int \mathrm{d}\Phi_R \sum_{\{ij\}} \mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R) \left[O(\Phi_R) - O(b_{ij}(\Phi_R)) \right]$$

- Explicit correction term due to observable kinematics: $\langle O \rangle^{(\mathrm{corr})}$
- Essence of NLO+PS
  - Ignore $\langle O \rangle^{(\mathrm{corr})}$ for the time being
  - Apply PS resummation to first line using $\Delta^{(\mathrm{A})}$ in which $\mathcal{D}^{(\mathrm{PS})} \to \mathcal{D}^{(\mathrm{A})}$

Master formula for NLO+PS up to first emission

$$\langle O\rangle^{(\text{NLO+PS})} = \sum_{\vec{f}_B}\int \mathrm{d}\Phi_B\,\bar{\mathcal{B}}^{(\text{A})}(\Phi_B)\Bigg[\underbrace{\Delta^{(\text{A})}(t_0)}_{\text{unresolved}}\,O(\Phi_B)$$

$$+\sum_{\{\widetilde{ij}\}}\sum_{f_i}\int_{t_0}\mathrm{d}\Phi_{R|B}^{ij}\,\underbrace{\frac{\mathcal{D}_{ij}^{(\text{A})}(r_{\widetilde{ij}}(\Phi_B))}{\mathcal{B}(\Phi_B)}\,\Delta^{(\text{A})}(t)}_{\text{resolved, singular}}\,O(r_{\widetilde{ij}}(\Phi_B))\Bigg]$$

$$+\sum_{\vec{f}_R}\int \mathrm{d}\Phi_R\,\underbrace{\left[\mathcal{R}(\Phi_R)-\sum_{ij}\mathcal{D}_{ij}^{(\text{A})}(\Phi_R)\right]}_{\text{resolved, non-singular}}\,O(\Phi_R)$$

- This is generated in the following way:
  - Generate seed event according to first or second line of $\langle O\rangle^{(\text{NLO})}$ on last slide
  - Second line: $\mathbb{H}$-event with Φ_R is kept as-is $\rightarrow$ resolved, non-singular term
  - First line: $\mathbb{S}$-event with Φ_B is processed through one-step PS with $\Delta^{(\text{A})}$ $\Rightarrow$ emission (resolved, singular) or no emission (unresolved) above t_0
- To $\mathcal{O}(\alpha_s)$ this reproduces $\langle O\rangle^{(\text{NLO})}$ including the correction term
- Resolved cases: Subsequent emissions can be generated by ordinary PS

Two Options:

Original MC@NLO

Frixione, Webber (2002)

Choose the parton shower splitting kernels as additional subtraction terms:

$$\mathcal{D}_{ij}^{(\mathrm{A})} \to \mathcal{D}_{ij}^{(\mathrm{PS})}$$

- Exponentiation in "resolved, singular" contribution is naturally bounded by μ_F
- Problems with soft divergences in "resolved, non-singular" integration

$$\int \mathrm{d}\Phi_R \left[\mathcal{R}(\Phi_R) - \sum_{ij} \mathcal{D}_{ij}^{(\mathrm{PS})}(\Phi_R) \right]$$

- Workaround: Supplement $\mathcal{D}^{(\mathrm{PS})}$ with "soft suppression function" $\mathcal{G}$
- Since $\mathcal{G}$ is not exponentiated, NLO accuracy breaks down for sub-leading colour configurations

SHERPA's variant

Höche, Krauss, Schönherr, FS (2011)

Choose the full Catani-Seymour dipoles as additional subtraction terms:

$$\mathcal{D}_{ij}^{(\mathrm{A})} \to \mathcal{D}_{ij}^{(\mathrm{S})}$$

- $\bar{\mathcal{B}}^{(\mathrm{A})}$ simplified significantly
- $\mathcal{D}^{(\mathrm{S})}$ can become negative $\Rightarrow \Delta > 1$
- Generated in Sherpa by weighted $N_C = 3$ one-step PS based on subtraction terms $\mathcal{D}^{(\mathrm{S})}$
- Exact NLO accuracy also for sub-leading colour configurations
- Phase space boundary for exponentiation is imposed by cuts in dipole terms

Event generation setup

- SHERPA's MC@NLO for $W + 0$, $W + 1$, $W + 2$ and $W + 3$-jet production
- Virtual corrections from BLACKHAT, leading-colour approximation for the $W + 3$-jet virtual
- For $n > 0$ regularise requiring k_T jets with $p_\perp > 10$ GeV
- Exponentiation region restricted using $\alpha = 0.01$-cut in dipole terms Nagy (2003) (cf. outlook)
- CTEQ6.6 NLO PDF
- $\mu_R = \mu_F = 1/2\,\hat{H}'_T$, where $\hat{H}'_T = \sqrt{\sum p_{T,j}^2 + E_{T,W}^2}$.
- Three levels of event simulation:

"NLO"	Fixed-order
"MC@NLO 1em"	MC@NLO including hardest emission
"MC@NLO PL"	MC@NLO including full PS

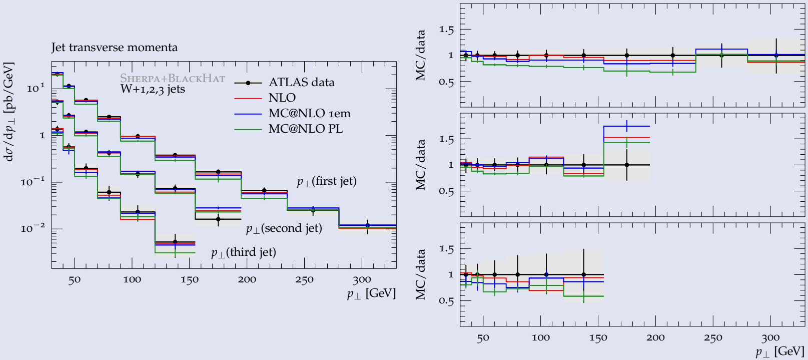
Analysis setup

- Comparing to ATLAS W+jets measurement arXiv:1201.1276
- Using implementation in Rivet arXiv:1003.0694
- Lepton with $p_\perp > 20$ GeV, $|\eta| < 2.5$
- $E_T^{\text{miss}} > 25$ GeV
- $m_{\text{T}}^{\text{W}} > 40$ GeV
- Anti-k_t jets with $R = 0.4$ and $p_\perp > 30$ GeV

Jet multiplicities

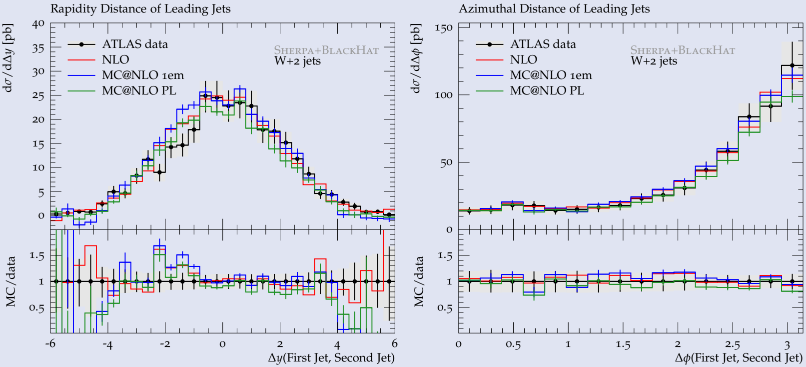
$W^{\pm} + \geq n$ jets	ATLAS	NLO	MC@NLO 1em	MC@NLO PL
$n = 0$	5.2 ± 0.2	5.06(1)	5.09(3)	5.06(3)
$n = 1, p_{\perp\, j} > 20$ GeV	0.95 ± 0.10	0.958(5)	0.968(10)	0.889(10)
$p_{\perp\, j} > 30$ GeV	0.54 ± 0.05	0.527(4)	0.534(7)	0.474(7)
$n = 2, p_{\perp\, j} > 20$ GeV	0.26 ± 0.04	0.263(2)	0.260(5)	0.236(4)
$p_{\perp\, j} > 30$ GeV	0.12 ± 0.02	0.120(1)	0.123(2)	0.109(2)
$n = 3, p_{\perp\, j} > 20$ GeV	0.068 ± 0.014	0.072(3)	0.059(3)	0.060(3)
$p_{\perp\, j} > 30$ GeV	0.026 ± 0.005	0.026(1)	0.022(2)	0.021(1)

Transverse momenta of jets



Transverse momentum of the first, second and third jet (from top to bottom) in $W^\pm + \geq 1, 2, 3$ jet production as measured by ATLAS compared to predictions from the corresponding fixed order and MC@NLO simulations.

Angular correlations of leading jets



Angular correlations of the two leading jets in $W^{\pm} + \geq 2$ jet production as measured by ATLAS compared to predictions from the $W^{\pm} + 2$ jet fixed order and MC@NLO simulations.

Summary

- NLO+PS matching was presented in common formalism
- MC@NLO developed as special case
- Colour-correctness achieved by exponentiating Catani-Seymour subtraction terms
- First NLO+PS predictions for W+3 jets
- Good agreement with experimental data from ATLAS

Outlook

- Improved functional form of dipole cut α will allow for better limitation of exponentiation region
- Merging NLO+PS with higher-multiplicity tree-level MEs can provide better description of multi-jet final states ($\rightarrow$ e.g. MENLOPS)
- Ultimate goal: Merging of NLO at different multiplicities + parton shower

Backup

Original POWHEG

- Choose additional subtraction terms as

$$\mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R) \to \rho_{ij}(\Phi_R)\,\mathcal{R}(\Phi_R) \qquad \text{where} \qquad \rho_{ij}(\Phi_R) = \frac{\mathcal{D}_{ij}^{(\mathrm{S})}(\Phi_R)}{\sum_{mn}\mathcal{D}_{mn}^{(\mathrm{S})}(\Phi_R)}$$

- $\mathbb{H}$-term vanishes
- $\bar{\mathcal{B}}^{(\mathrm{A})}$ remains complicated now, includes real-emission integration (may be done by Monte-Carlo method)
- Similar to PS with ME-correction for 1st emission (e.g. Herwig, Pythia)

Mixed scheme

- Subtract arbitrary regular piece from $\mathcal{R}$ and generate separately

$$\mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R) \to \rho_{ij}(\Phi_R)\,\left[\mathcal{R}(\Phi_R) - \mathcal{R}^r(\Phi_R)\right] \qquad \text{where} \qquad \rho_{ij} \text{ as above}$$

- Allows to generate the non-singular cases of $\mathcal{R}$ without underlying $\mathcal{B}$
- More control over how much is exponentiated