NLO accuracy in modern Monte-Carlo event generators

Graduate School Freiburg, 8 May 2013

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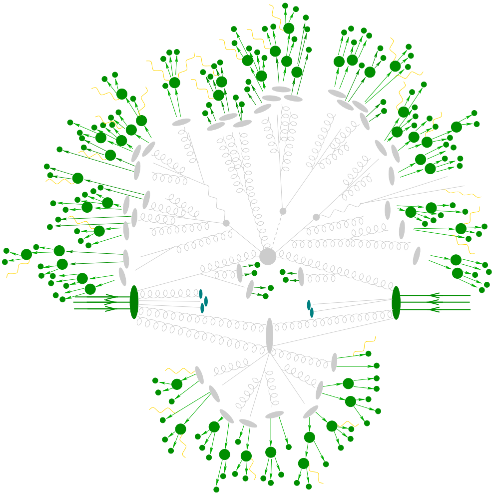
Introduction: Monte-Carlo event generators



- We want:
  Simulation of $pp \to$ full hadronised final state
- MC event representation for $pp \to t\bar{t}H$
- We know from first principles:
  - Hard scattering at fixed order in perturbation theory (**Matrix Element**)
  - Approximate resummation of QCD corrections to all orders (**Parton Shower**)
- Missing bits: Hadronisation/Underlying event (ignored here)

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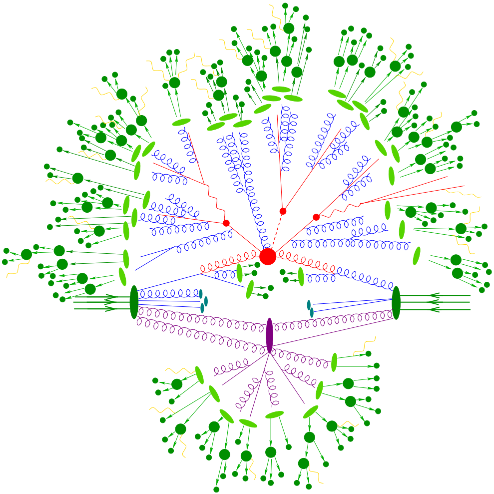

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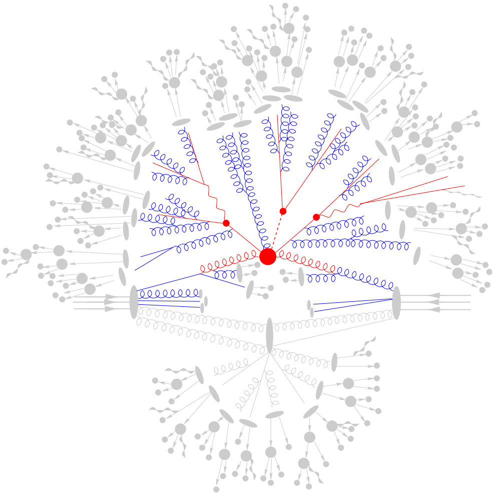

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Introduction: Monte-Carlo event generators

Outline

- Reminder: Perturbation theory for QCD
- The parton shower approximation
- Correcting that approximation as far as possible:
  - NLO+PS (2002)
  - Tree-level ME+PS merging (2001)
  - MENLOPS (2010)
  - ME+PS merging at NLO (2012)

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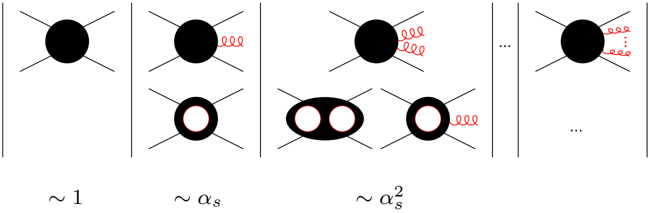
Perturbation Theory

- Cannot solve QCD and calculate e.g. $pp \to t\bar{t}H$ exactly
- But can calculate parts of the perturbative series in α_s:



- Exact calculations possible up to $\mathcal{O}(\alpha_s^2)$ for some processes
- $\alpha_s^2 \approx 1\% \Rightarrow$ high enough precision, right?
- Why is that not always true?

From fixed order to resummation

- Predictions for inclusive observables calculable at fixed-order ($\rightsquigarrow$ KLN theorem)
- But what if not inclusive enough, e.g.:
  - Study certain regions of phase space, like $p_\perp^Z \to 0$ @ DY
  - Making predictions for hadron-level final states: confinement at $\mu_{\text{had}} \approx 1$ GeV

  $\Rightarrow$ Finite remainders of infrared divergences:

  $$\text{logarithms of } \frac{\mu^2_{\text{hard}}}{\mu^2_{\text{res}}} \text{ with each } \mathcal{O}(\alpha_s)$$

  are large and spoil convergence of perturbative series
- Need to resum the series to all orders
  - Problem: We are not smart enough for that.
  - Workaround: Resum only the logarithmically enhanced terms in the series
- Parton showers resum these terms in their evolution of a parton ensemble between μ^2_{hard} and μ^2_{had}

  How?

Construction of a parton shower (PS)

- Evolution of parton ensemble simulated by recursive parton branchings
- Probability for branching at each step includes (resums) arbitrarily many earlier branchings

Let's start simple: one emission, no resummation

- Universal factorisation of QCD real emission ME for collinear parton pair (i, j):

$$\mathcal{R} \;\to\; \mathcal{D}_{ij}^{(\mathrm{PS})} = \mathcal{B} \times \left[\frac{1}{2p_ip_j}\; 8\pi\alpha_s\; \mathcal{K}_{ij}(p_i, p_j)\right]$$

  - $\mathcal{B}$ = Born matrix element
  - $\mathcal{K}_{ij}$ splitting kernel for branching $(ij) \to i + j$
    Specific form depends on factorisation scheme (DGLAP, Catani-Seymour, Antenna, ...)
  - Massless propagator $\frac{1}{2p_ip_j}$
    (Later: Evolution variable of shower $t \sim 2p_ip_j$, e.g. $k_\perp$, angle, ...)

- Radiative phase space factorises as well: $\mathrm{d}\Phi_{\mathcal{R}} = \mathrm{d}\Phi_{\mathcal{B}}\mathrm{d}\Phi_1 = \mathrm{d}\Phi_{\mathcal{B}}\; \mathrm{d}t \frac{1}{16\pi^2}\; \mathrm{d}z\; \frac{\mathrm{d}\phi}{2\pi}$
  (ignoring z and ϕ dependence from here on, because they are "trivial", not related to large logs)

- Combined with radiative part of the factorised ME (Jacobian/symmetry factor/PDFs ignored)

$$\mathrm{d}\sigma_{ij}^{(\mathrm{PS})} \sim \mathrm{d}t\,\mathcal{D}_{ij}^{(\mathrm{PS})} \sim \frac{\mathrm{d}t}{t}\,\frac{\alpha_s}{2\pi}\,\mathcal{K}_{ij}$$

Differential branching probability

Resummed branching probability

Evolution with respect to t

- $\mathrm{d}\sigma_{ij}^{(\mathrm{PS})} \sim \mathrm{d}t\, \mathcal{D}_{ij}^{(\mathrm{PS})}$ is universal and appears for each emission
- How do we get the resummed branching probability according to multiple such emissions?

$\rightarrow$ Analogy to evolution of ensemble of radioactive nuclei:
Survival probability at time t_1 depends on decay/survival at times $t < t_1$

Radioactive decay

- Constant differential decay probability

$$f(t) = \text{const} \equiv \lambda$$

- Survival probability $\mathcal{N}(t)$

$$-\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}t} = \lambda\, \mathcal{N}(t)$$

$$\Rightarrow \mathcal{N}(t) \sim \exp(-\lambda t)$$

- Resummed decay probability $\mathcal{P}(t)$

$$\mathcal{P}(t) = f(t)\, \mathcal{N}(t) \sim \lambda \exp(-\lambda t)$$

Parton shower branching

- Differential branching probability

$$f(t) \equiv \mathcal{D}_{ij}^{(\mathrm{PS})}$$

- Survival probability $\mathcal{N}(t)$

$$-\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}t} = f(t)\, \mathcal{N}(t)$$

$$\Rightarrow \mathcal{N}(t) \sim \exp\left(- \int_0^t f(t')\mathrm{d}t' \right)$$

- Resummed branching probability $\mathcal{P}(t)$

$$\mathcal{P}(t) = f(t)\, \mathcal{N}(t) \sim f(t) \exp\left(- \int_0^t f(t')\mathrm{d}t' \right)$$

Algorithmic implementation

Parton shower algorithm

- Recursively generates next emission scale t (after t_{previous}) with probability

$$\mathcal{P}(t, t_{\text{previous}}) = f(t) \exp\left(- \int_{t_{\text{previous}}}^{t} f(t')\mathrm{d}t' \right)$$

- Analytically: $t = F^{-1}\left[F(t_{\text{previous}}) + \log(\#_{\text{random}})\right]$ with $F(t) = \int_{t_0}^{t} \mathrm{d}t' f(t')$
- If integral/its inverse are not known: "Veto algorithm" = extension of hit-or-miss
  - Overestimate $g(t) \geq f(t)$ with known integral $G(t)$ $\rightarrow t = G^{-1}\left[G(t_{\text{previous}}) + \log(\#_{\text{random}})\right]$
  - Accept t with probability $\frac{f(t)}{g(t)}$ using hit-or-miss

Definition of main parton shower ingredients

- "Sudakov form factor" ≡ Survival probability of parton ensemble between two scales:

$$\Delta(t', t'') = \prod_{\{ij\}} \exp\left(- \int_{t'}^{t''} \mathrm{d}t \, \mathcal{D}_{ij}^{(\mathrm{PS})} \right)$$

- Evolution variable t: not time, but scale of collinearity from hard to soft
  $t \sim 2p_i p_j \sim$ e.g. angle θ, virtuality Q^2, relative transverse momentum $k_\perp^2$, ...
- Starting scale μ_Q^2 (time $t = 0$ in radioactive decay) defined by hard scattering
- Cutoff scale related to hadronisation scale $t_0 \sim \mu_{\mathrm{had}}^2$
- Other variables (z, ϕ) generated directly according to $\mathrm{d}\sigma_{ij}^{(\mathrm{PS})}(t, z, \phi)$

⇒ Differential cross section (up to first emission)

$$\mathrm{d}\sigma = \mathrm{d}\Phi_B \, \mathcal{B} \left[\underbrace{\Delta^{(\mathrm{PS})}(t_0, \mu_Q^2)}_{\text{unresolved}} + \underbrace{\sum_{\{ij\}} \int_{t_0}^{\mu_Q^2} \mathrm{d}t \, \frac{\mathrm{d}\sigma_{ij}^{(\mathrm{PS})}}{\mathrm{d}t} \, \Delta^{(\mathrm{PS})}(t, \mu_Q^2)}_{\text{resolved}} \right]$$

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Improving parton showers at fixed order: Classification

NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive W production)
- Objectives:
  - avoid double counting in real emission
  - preserve inclusive NLO accuracy

ME+PS@LO merging

- Multiple LO+PS simulations for processes of different jet multi (e.g. $W, Wj, Wjj, \ldots$)
- Objectives:
  - combine into one inclusive sample by making them exclusive
  - preserve resummation accuracy

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Combination: ME+PS@NLO

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Reminder + Notation: Subtraction method

- Contributions to NLO cross section: $\mathcal{B}$orn, $\mathcal{V}$irtual and $\mathcal{R}$eal emission
- $\mathcal{V}$ and $\mathcal{R}$ divergent in separate phase space integrations
  $\Rightarrow$ Subtraction terms $\mathcal{D}$ and their integrated form $\mathcal{I}$ for NLO cross section:

$$\mathrm{d}\sigma^{(\mathrm{NLO})} = \mathrm{d}\Phi_B \left[\mathcal{B} + \tilde{\mathcal{V}} + \sum_{\{ij\}} \mathcal{I}^{(\mathrm{S})}_{(ij)} \right] + \mathrm{d}\Phi_R \left[\mathcal{R} - \sum_{\{ij\}} \mathcal{D}^{(\mathrm{S})}_{ij} \right]$$

Idea of NLO+PS matching

- Applying PS resummation to LO event was "simple" ✓
- Apply the same separately for $\mathcal{B}$ and $\mathcal{V}+\mathcal{I}$ and $\mathcal{R}-\mathcal{D}$ at NLO? ✗
  $\Rightarrow$ double counting
- Instead: additional subtraction terms $\mathcal{D}^{(\mathrm{A})}_{ij}$

$$\mathrm{d}\sigma^{(\mathrm{NLO\,sub})} = \mathrm{d}\Phi_B\ \bar{\mathcal{B}}^{(\mathrm{A})} + \mathrm{d}\Phi_R \left[\mathcal{R} - \sum_{\{ij\}} \mathcal{D}^{(\mathrm{A})}_{ij} \right]$$

$$\text{with } \bar{\mathcal{B}}^{(\mathrm{A})} = \mathcal{B} + \tilde{\mathcal{V}} + \sum_{\{ij\}} \mathcal{I}^{(\mathrm{S})}_{(ij)} + \sum_{\{ij\}} \int \mathrm{d}t \left[\mathcal{D}^{(\mathrm{A})}_{ij} - \mathcal{D}^{(\mathrm{S})}_{ij} \right]$$

- Now apply PS resummation to $\mathrm{d}\sigma^{(\mathrm{NLO\,sub})}$ events, using $\mathcal{D}^{(\mathrm{A})}_{ij}$ as splitting kernels
  $\rightarrow$ reproduces $\mathrm{d}\sigma^{(\mathrm{NLO})} + \mathcal{O}(\alpha_s^2)$

Master formula for NLO+PS up to first emission

$$\mathrm{d}\sigma^{(\mathrm{NLO+PS})} = \mathrm{d}\Phi_B\, \bar{\mathcal{B}}^{(\mathrm{A})} \Bigg[\underbrace{\Delta^{(\mathrm{A})}(t_0, \mu_Q^2)}_{\text{unresolved}} + \underbrace{\sum_{\{ij\}} \int_{t_0}^{\mu_Q^2} \mathrm{d}t\, \frac{\mathcal{D}_{ij}^{(\mathrm{A})}}{\mathcal{B}} \Delta^{(\mathrm{A})}(t, \mu_Q^2)}_{\text{resolved, singular}} \Bigg]$$

$$+ \ \mathrm{d}\Phi_R \ \underbrace{\Bigg[\mathcal{R} - \sum_{\{ij\}} \mathcal{D}_{ij}^{(\mathrm{A})} \Bigg]}_{\text{resolved, non-singular} \equiv \mathcal{H}^{(\mathrm{A})}}$$

- To $\mathcal{O}(\alpha_s)$ this reproduces $\mathrm{d}\sigma^{(\mathrm{NLO})}$ including the correction term
- Event generation: $\bar{\mathcal{B}}^{(\mathrm{A})}$ or $\mathcal{H}^{(\mathrm{A})}$ seed event according to their XS
  - First line ("$\mathbb{S}$-event"): from one-step PS with $\Delta^{(\mathrm{A})}$
    $\Rightarrow$ emission (resolved, singular) or no emission (unresolved) above t_0
  - Second line ("$\mathbb{H}$-event"): kept as-is $\to$ resolved, non-singular term
- Resolved cases: Subsequent emissions can be generated by ordinary PS
- Exact choice of $\mathcal{D}_{ij}^{(\mathrm{A})}$ will specify MC@NLO vs. POWHEG

Special case: MC@NLO

To prove NLO accuracy:
$\mathcal{D}^{(\mathrm{A})}$ needs to be identical in shower algorithm and $\mathbb{H}$-events

Original idea: $\mathcal{D}^{(\mathrm{A})}$ = PS splitting kernels

Frixione, Webber (2002)

+ Shower algorithm for Born-like events easy to implement
– "Non-singular" piece $\mathcal{R} - \sum_{ij} \mathcal{D}_{ij}^{(\mathrm{A})}$ is actually singular:
  - Collinear divergences subtracted by splitting kernels ✓
  - Remaining soft divergences as they appear in non-trivial processes at sub-leading N_c ✗

Workaround: $\mathcal{G}$-function dampens soft limit in non-singular piece
⇔ Loss of formal NLO accuracy
(but heuristically only small impact)

Alternative idea: $\mathcal{D}^{(\mathrm{A})}$ = Catani-Seymour dipole subtraction terms $\mathcal{D}^{(\mathrm{S})}$

(only potential difference: phase space cuts)

Höche, Krauss, Schönherr, FS (2011)

+ "Non-singular" piece fully free of divergences
– Splitting kernels in shower algorithm become negative

Solution: Weighted $N_C = 3$ one-step PS based on subtraction terms

⇓

Used in SHERPA

Special case: POWHEG

Original POWHEG

- Choose additional subtraction terms as

$$\mathcal{D}_{ij}^{(\mathrm{A})} \to \rho_{ij}\,\mathcal{R} \qquad \text{where} \qquad \rho_{ij} = \frac{\mathcal{D}_{ij}^{(\mathrm{S})}}{\sum_{mn}\mathcal{D}_{mn}^{(\mathrm{S})}}$$

- $\mathcal{H}$-term vanishes $\Rightarrow$ No negative weighted events
- Similar to PS with ME-correction for 1st emission (e.g. Herwig, Pythia)

Mixed scheme

- Subtract arbitrary regular piece from $\mathcal{R}$ and generate separately as $\mathbb{H}$-events

$$\mathcal{D}_{ij}^{(\mathrm{A})}(\Phi_R) \to \rho_{ij}(\Phi_R)\,\left[\mathcal{R}(\Phi_R) - \mathcal{R}^r(\Phi_R)\right] \qquad \text{where} \qquad \rho_{ij} \text{ as above}$$

- Tuning of $\mathcal{R}^r$ to reduce exponentiation of arbitrary terms
- Allows to generate the non-singular cases of $\mathcal{R}$ without underlying $\mathcal{B}$

Inherent systematic uncertainties

Perturbative uncertainties

- Unknown higher-order corrections
- Estimated by scale variations
  $\mu_F = \mu_R = \frac{1}{2}\mu \ldots 2\mu$

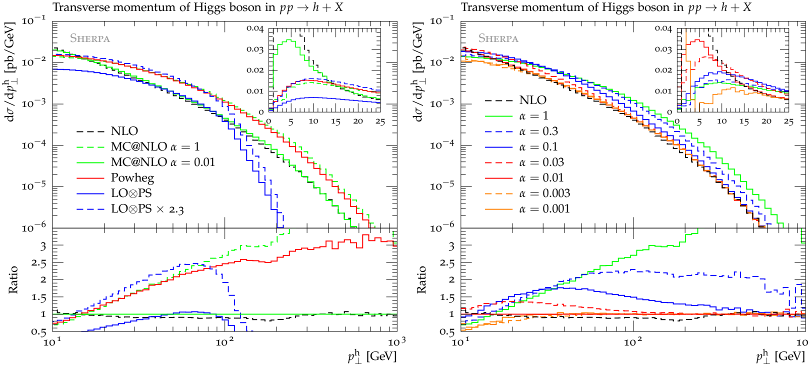
Non-perturbative uncertainties

- Model uncertainties in hadronisation, hadron decays, multiple parton interactions
- Estimated by variation of parameters/models within tuned ranges

Exponentiation uncertainties

- Arbitrariness of $\mathcal{D}^{(\mathrm{A})}$ and thus of the exponent in $\Delta^{(\mathrm{A})}$
- Estimated by:
  - Variations of μ_Q^2 in MC@NLO
  - (Variation of $\mathcal{R}^r$ in POWHEG)
- Reduced by merging with NLO for higher parton multiplicities $\rightsquigarrow$ later

Case study: Higgs production in gluon-gluon-fusion



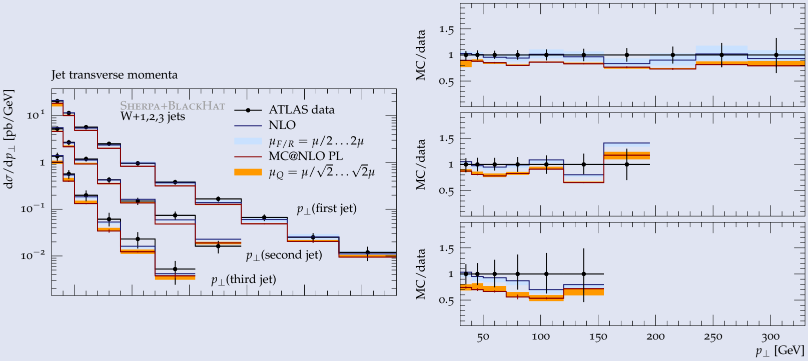
For demonstration purposes

- Strong sensitivity to exponentation especially at large $p_\perp^h$
- POWHEG and completely unrestricted MC@NLO similar
- Decrease exponentiation of non-singular pieces using unphysical dipole α:
  $\alpha_{\text{cut}} \lesssim 0.01$ recovers NLO behaviour

State-of-the-art application: W+3-jet production

Proper physical assessment of variation:
Dipole restriction at (and variation of) resummation scale μ_Q

Comparison to ATLAS data



ATLAS measurement (arXiv:1201.1276)
SHERPA+BLACKHAT NLO+PS predictions (arXiv:1201.5882)

Improving parton showers at fixed order: Classification

NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive W production)
- Objectives: ✓
  - avoid double counting in real emission
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ME+PS@LO merging

- Multiple LO+PS simulations for processes of different jet multi (e.g. $W, Wj, Wjj, \ldots$)
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Tree-level ME+PS merging

Main idea

Phase space slicing for QCD radiation in shower evolution

- Hard emissions $Q_{ij}(z,t) > Q_{\text{cut}}$
  - Events rejected
  - Compensated by events starting from higher-order ME regularised by Q_{cut}

  $\Rightarrow$ Splitting kernels replaced by exact real-emission matrix elements

$$\mathcal{D}_{ij}^{(\text{PS})} \quad \rightarrow \quad \mathcal{R}_{ij}$$

(But Sudakov form factors $\Delta^{(\text{PS})}$ remain unchanged)

- Soft/collinear emissions $Q_{ij,k}(z,t) < Q_{\text{cut}}$

  $\Rightarrow$ Retained from parton shower $\quad \mathcal{D}_{ij}^{(\text{PS})} = \mathcal{B} \times \left[\frac{1}{2p_i p_j}\, 8\pi\alpha_s\, \mathcal{K}_{ij}(p_i, p_j)\right]$

Note

Boundary determined by "jet criterion" $Q_{ij,k}$

- Has to identify soft/collinear divergences in MEs, like jet algorithm
- Otherwise arbitrary

Parton shower on top of high-multi ME

Translate ME event into shower language

Why?

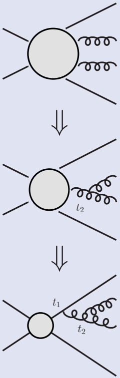
- Need starting scales t for PS evolution
- Have to embed existing emissions into PS evolution

Problem: ME only gives final state, no history

Solution: Backward-clustering (running the shower reversed), similar to jet algorithm:

1. Select last splitting according to shower probablities
2. Recombine partons using inverted shower kinematics $\rightarrow$ N-1 particles + splitting variables for one node
3. Reweight $\alpha_s(\mu^2) \rightarrow \alpha_s(p_\perp^2)$
4. Repeat 1 - 3 until core process ($2 \rightarrow 2$)

Example:



Truncated shower

- Shower each (external and intermediate!) line between determined scales
- "Boundary" scales: resummation scale μ_Q^2 and shower cut-off t_0

Master formula

Cross section up to first emission in ME+PS

$$\mathrm{d}\sigma = \mathrm{d}\Phi_B\, \mathcal{B}\Bigg[\underbrace{\Delta^{(\mathrm{PS})}(t_0, \mu_Q^2)}_{\text{unresolved}} + \sum_{\{ij\}} \int_{t_0}^{\mu_Q^2} \mathrm{d}t\, \Delta^{(\mathrm{PS})}(t, \mu^2) \times \Bigg(\underbrace{\frac{\mathcal{D}_{ij}^{(\mathrm{PS})}}{\mathcal{B}}\, \Theta(Q_{\mathrm{cut}} - Q_{ij})}_{\text{resolved, PS domain}} + \underbrace{\frac{\mathcal{R}_{ij}}{\mathcal{B}}\, \Theta(Q_{ij} - Q_{\mathrm{cut}})}_{\text{resolved, ME domain}} \Bigg)\Bigg]$$

Features

- LO weight B for Born-like event
- Unitarity slightly violated due to mismatch of $\Delta^{(\mathrm{PS})}$ and $\mathcal{R}/\mathcal{B}$ $[\ldots] \approx 1 \Rightarrow$ LO cross section only approximately preserved
- Unresolved emissions as in parton shower approach
- Resolved emissions now sliced into PS and ME domain
- Only for one emission here, but possible up to high number of emissions

Features and shortcomings by example

Example

Diphoton production at Tevatron

- Measured by CDF Phys.Rev.Lett. 110 (2013) 101801
- Isolated hard photons
- Azimuthal angle between the diphoton pair

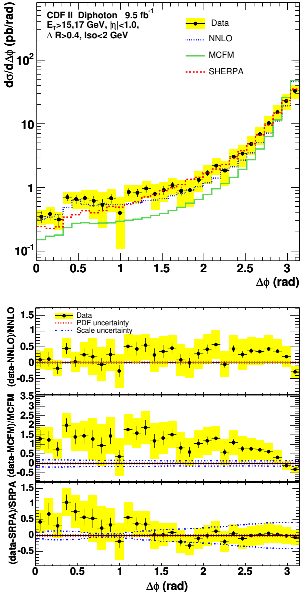
ME+PS simulation using SHERPA vs. (N)NLO

Conclusions

Shapes described very well even for this non-trivial process/observable for both:

- Hard region, e.g. $\Delta\Phi_{\gamma\gamma} \to 0$
- Soft region, e.g. $\Delta\Phi_{\gamma\gamma} \to \pi$

Scale variations high $\Rightarrow$ NLO needed



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  - combine into one inclusive sample
  - preserve NLO accuracy for jet observables

Basic idea

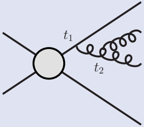
Concepts continued from ME+PS merging at LO

- For each event select jet multiplicity k according to its inclusive NLO cross section
- Reconstruct branching history and nodal scales $t_0 \dots t_k$
- Truncated vetoed parton shower, but with peculiarities (cf. below)

Differences for NLO merging

- For each event select type ($\mathbb{S}$ or $\mathbb{H}$) according to absolute XS
  $\Rightarrow$ Shower then runs differently
- $\mathbb{S}$ event:
  1. Generate MC@NLO emission at t_{k+1}
  2. Truncated "NLO-vetoed" shower between t_0 and t_k:
     First hard emission is only ignored, no event veto
  3. Continue with vetoed parton shower
- $\mathbb{H}$ event:
  (Truncated) vetoed parton shower as in tree-level ME+PS

Example: $k = 1$

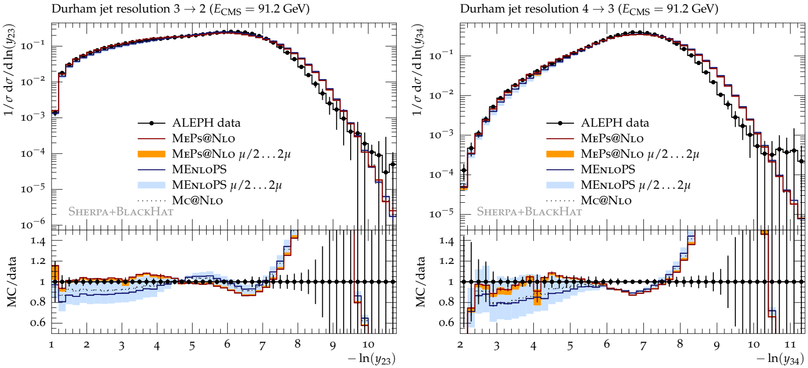


Master formula

ME+PS@NLO prediction for combining NLO+PS samples of multiplicities n and $n+1$

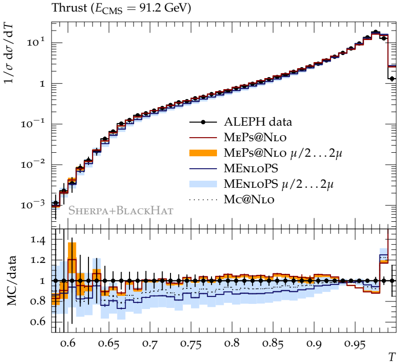
$$
\begin{aligned}
\mathrm{d}\sigma \;=\; & \mathrm{d}\Phi_n\,\bar{\mathrm{B}}_n^{(\mathrm{A})}\left[\Delta_n^{(\mathrm{A})}(t_c,\mu_Q^2)+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\,\frac{\mathrm{D}_n^{(\mathrm{A})}}{\mathrm{B}_n}\,\Delta_n^{(\mathrm{A})}(t_{n+1},\mu_Q^2)\,\Theta(Q_{\mathrm{cut}}-Q_{n+1})\right] \\
& + \mathrm{d}\Phi_{n+1}\,\mathrm{H}_n^{(\mathrm{A})}\,\Delta_n^{(\mathrm{PS})}(t_{n+1},\mu_Q^2)\,\Theta(Q_{\mathrm{cut}}-Q_{n+1}) \\
& + \mathrm{d}\Phi_{n+1}\,\bar{\mathrm{B}}_{n+1}^{(\mathrm{A})}\underbrace{\left(1+\frac{\mathrm{B}_{n+1}}{\bar{\mathrm{B}}_{n+1}^{(\mathrm{A})}}\int_{t_{n+1}}^{\mu_Q^2}\mathrm{d}\Phi_1\,\mathrm{K}_n\right)}_{\text{MC counterterm} \,\to\, \text{NLO-vetoed shower}}\Delta_n^{(\mathrm{PS})}(t_{n+1},\mu_Q^2)\,\Theta(Q_{n+1}-Q_{\mathrm{cut}}) \\
& \times\left[\Delta_{n+1}^{(\mathrm{A})}(t_c,t_{n+1})+\int_{t_c}^{t_{n+1}}\mathrm{d}\Phi_1\,\frac{\mathrm{D}_{n+1}^{(\mathrm{A})}}{\mathrm{B}_{n+1}}\,\Delta_{n+1}^{(\mathrm{A})}(t_{n+2},t_{n+1})\right] \\
& + \mathrm{d}\Phi_{n+2}\,\mathrm{H}_{n+1}^{(\mathrm{A})}\,\Delta_{n+1}^{(\mathrm{PS})}(t_{n+2},t_{n+1})\,\Delta_n^{(\mathrm{PS})}(t_{n+1},\mu_Q^2)\,\Theta(Q_{n+1}-Q_{\mathrm{cut}})\;+\;\ldots
\end{aligned}
$$

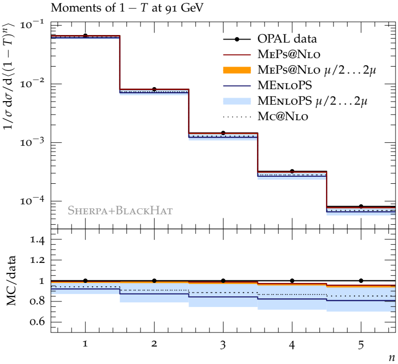
Results for $e^+e^- \to$ hadrons: Differential Durham jet rates



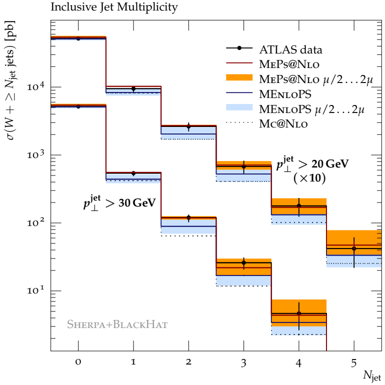
- Significant reduction of ME+PS@NLO scale uncertainties in perturbative region
- Improved agreement with experimental data

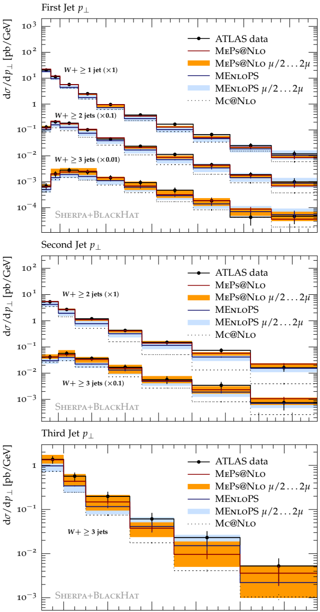
Results for $e^+e^- \to$ hadrons: Thrust event shape





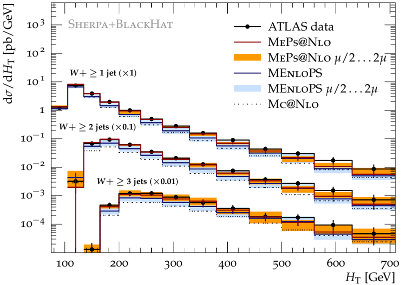
Results for W + jets: Jet multiplicities and $p_\perp$

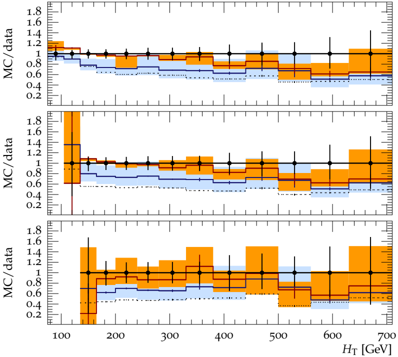




- Comparison to ATLAS measurement Phys.Rev. D85 (2012), 092002
- Significant reduction of ME+PS@NLO scale uncertainties in "NLO" multiplicities
- Improved agreement with data

Results for W + jets: Scalar transverse momentum sum H_T

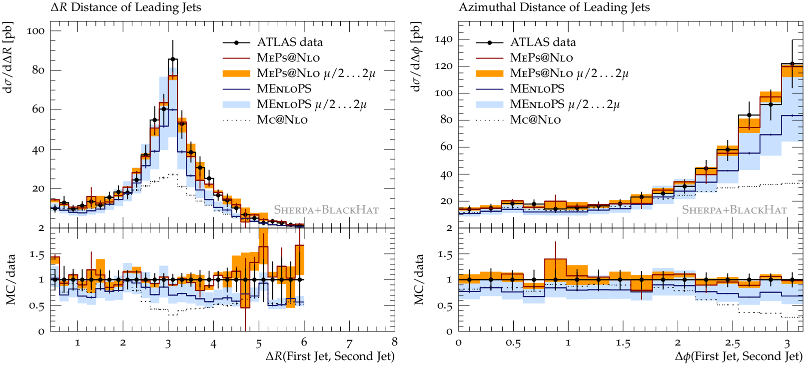




H_T and related observables are sensitive to many jet multiplicities simultaneously

- Need ME+PS@NLO for precise description
- High H_T region affected by higher multiplicities ⇒ Larger scale uncertainty

Results for W + jets: Angular correlations



- Pure MC@NLO simulation misses correlations between the two leading jets

Conclusions

NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive W production)
- Objectives: ✓
  - avoid double counting in real emission
  - preserve inclusive NLO accuracy

ME+PS@LO merging

- Multiple LO+PS simulations for processes of different jet multi (e.g. $W, Wj, Wjj, \ldots$)
- Objectives: ✓
  - combine into one inclusive sample by making them exclusive
  - preserve resummation accuracy

$\Downarrow$ $\Downarrow$

Combination: ME+PS@NLO

- Multiple NLO+PS simulations for processes of different jet multiplicity e.g. $W, Wj, Wjj, \ldots$
- Objectives: ✓ ✓
  - combine into one inclusive sample
  - preserve NLO accuracy for jet observables

Conclusions

Summary

- I'm most certainly out of time by now.

Outlook

- Wine and cheese.

Backup material

General setup

- ME generators (tree-level and dipole subtraction): AMEGIC++ and COMIX
- Virtual corrections from BLACKHAT
- MC@NLO-like generator built into SHERPA with full colour treatment
- Parton shower based on Catani-Seymour dipole factorisation
- Hadronisation model AHADIC++, not tuned for ME+PS@NLO yet
  $\Rightarrow$ Deviations in hadronisation sensitive regions
- Comparison to ALEPH and OPAL measurements:
  Eur. Phys. J. C35 (2004), 457-486, Eur.Phys.J. C40 (2005), 287-316, Eur. Phys. J. C20 (2001), 601-615

Comparison of three runs

MC@NLO: NLO+PS prediction for $2 \to 2$

MENLOPS: MC@NLO for $2 \to 2$ + ME+PS up to $2 \to 6$
μ_R variation indicated by blue band

ME+PS@NLO: MC@NLO for $2 \to 2, 3, 4$ + ME+PS for $2 \to 5, 6$
μ_R variation indicated by orange band

General setup

- ME generators (tree-level and dipole subtraction): AMEGIC++ and COMIX
- Virtual corrections from BLACKHAT
- MC@NLO-like generator built into SHERPA with full colour treatment
- Parton shower based on Catani-Seymour dipole factorisation
- Hadronisation and multiple parton interactions not taken into account (observables almost insensitive)
- CT10 PDF set
- Central scales $\mu_{F,R}$ from clustering onto $2 \to 2$ configuration

Comparison of three runs

MC@NLO: NLO+PS prediction for $2 \to 2$

MENLOPS: MC@NLO for $2 \to 2$ + ME+PS up to $2 \to 6$
$\mu_{F,R}$ variation indicated by blue band

ME+PS@NLO: MC@NLO for $2 \to 2, 3, 4$ + ME+PS for $2 \to 5, 6$
$\mu_{F,R}$ variation indicated by orange band