# CONSTRUCTING AND IMPROVING QCD PARTON SHOWER SIMULATIONS 

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- We want:

Simulation of $p p \rightarrow$ full hadronised final state

- MC event representation for $p p \rightarrow t \bar{t} H$ event
- We know from first principles:
- Hard scattering at fixed
order in perturbation
theory
(Matrix Element)
- Approximate
resummation of QCD corrections to all orders
(Tarton Shower)

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- Approximate resummation of QCD corrections to all orders (Parton Shower)
- Missing bits:

Hadronisation/Underlying event (ignored here)

## Outline

- Reminder: QCD perturbation theory
- The parton shower approximation
- Correcting that approximation as far as possible:
- NLO+PS (2002)
- Tree-level ME+PS merging (2001)
- Menlops (2010)
- ME+PS merging at NLO (2012)
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- We know from first principles:
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(Matrix Element)
- Approximate resummation of QCD corrections to all orders (Parton Shower)
- Cannot solve QCD and calculate e.g. $p p \rightarrow t \bar{t} H$ exactly
- But can calculate parts of the perturbative series in $\alpha_{s}$ :

- Exact calculations possible up to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ for some processes
- $\alpha_{s}^{2} \approx 1 \% \Rightarrow$ high enough precision, right?
- Why is that not always true?
- Predictions for inclusive observables calculable at fixed-order ( $\rightsquigarrow$ KLN theorem)
- But what if not inclusive enough, e.g.:
- Study certain regions of phase space, like $p_{\perp}^{Z} \rightarrow 0 @$ DY
- Making predictions for hadron-level final states: confinement at $\mu_{\text {had }} \approx 1 \mathrm{GeV}$
$\Rightarrow$ Finite remainders of infrared divergences:

$$
\text { logarithms of } \frac{\mu_{\text {hard }}^{2}}{\mu_{\text {res }}^{2}} \text { with each } \mathcal{O}\left(\alpha_{s}\right)
$$

are large and spoil convergence of perturbative series

- Need to resum the series to all orders
- Problem: We are not smart enough for that.
- Workaround: Resum only the logarithmically enhanced terms in the series
- Parton showers resum these terms in their evolution of a parton ensemble between $\mu_{\text {hard }}^{2}$ and $\mu_{\text {had }}^{2}$
How?


## Let's start simple: one emission, no resummation

- Universal factorisation of QCD real emission ME for collinear parton pair $(i, j)$ :

$$
\mathcal{R} \rightarrow \mathcal{D}_{i j}^{(\mathrm{PS})}=\mathcal{B} \times\left[\frac{1}{2 p_{i} p_{j}} 8 \pi \alpha_{s} \mathcal{K}_{i j}\left(p_{i}, p_{j}\right)\right]
$$

- $\mathcal{B}=$ Born matrix element
- $\mathcal{K}_{i j}$ splitting kernel for branching $(i j) \rightarrow i+j$

Specific form depends on factorisation scheme (DGLAP, Catani-Seymour, Antenna, ...)

- Massless propagator $\frac{1}{2 p_{i} p_{j}}$
(Later: Evolution variable of shower $t \sim 2 p_{i} p_{j}$, e.g. $k_{\perp}$, angle, $\ldots$ )
- Radiative phase space factorises as well:
$\mathrm{d} \Phi_{\mathcal{R}}=\mathrm{d} \Phi_{\mathcal{B}} \mathrm{d} \Phi_{1}=\mathrm{d} \Phi_{\mathcal{B}} \mathrm{d} t \frac{1}{16 \pi^{2}} \mathrm{~d} z \frac{\mathrm{~d} \phi}{2 \pi}$
(ignoring $z$ and $\phi$ dependence from here on, because they are "trivial", not related to large logs)
- Combined with radiative part of the factorised ME
(Jacobian/symmetry factor/PDFs ignored)
$\mathrm{d} \sigma_{i j}^{(\mathrm{PS})} \sim \mathrm{d} t \mathcal{D}_{i j}^{(\mathrm{PS})} \sim \frac{\mathrm{d} t}{t} \frac{\alpha_{s}}{2 \pi} \mathcal{K}_{i j} \quad$ Differential branching probability


## Evolution with respect to $t$

- $\mathrm{d} \sigma_{i j}^{(\mathrm{PS})} \sim \mathrm{d} t \mathcal{D}_{i j}^{(\mathrm{PS})}$ is universal and appears for each emission
- How do we get the resummed branching probability according to multiple such emissions?
$\rightarrow$ Analogy to evolution of ensemble of radioactive nuclei:
Survival probability at time $t_{1}$ depends on decay/survival at times $t<t_{1}$


## Radioactive decay

- Constant differential decay probability

$$
f(t)=\text { const } \equiv \lambda
$$

- Survival probability $\mathcal{N}(t)$

$$
\begin{aligned}
&-\frac{\mathrm{d} \mathcal{N}}{\mathrm{~d} t}=\lambda \mathcal{N}(t) \\
& \Rightarrow \mathcal{N}(t) \sim \exp (-\lambda t)
\end{aligned}
$$

## Parton shower branching

- Differential branching probability

$$
f(t) \equiv \mathcal{D}_{i j}^{(\mathrm{PS})}
$$

- Survival probability $\mathcal{N}(t)$

$$
\begin{gathered}
-\frac{\mathrm{d} \mathcal{N}}{\mathrm{~d} t}=f(t) \mathcal{N}(t) \\
\Rightarrow \mathcal{N}(t) \sim \exp \left(-\int_{0}^{t} \mathrm{~d} t^{\prime} f\left(t^{\prime}\right)\right)
\end{gathered}
$$

## Algorithmic PS implementation

## Radioactive decay

- Survival probability $\mathcal{N}(t)$
$\mathcal{N}(t) \sim \exp (-\lambda t)$
- Resummed decay probability $\mathcal{P}(t)$

$$
\mathcal{P}(t)=f(t) \mathcal{N}(t) \sim \lambda \exp (-\lambda t)
$$

## Parton shower branching

- Survival probability $\mathcal{N}(t)$
$\mathcal{N}(t) \sim \exp \left(-\int_{0}^{t} f\left(t^{\prime}\right) \mathrm{d} t^{\prime}\right)$
- Resummed branching probability $\mathcal{P}(t)$

$$
\mathcal{P}(t)=f(t) \mathcal{N}(t) \sim f(t) \exp \left(-\int_{0}^{t} \mathrm{~d} t^{\prime} f\left(t^{\prime}\right)\right)
$$

## Parton shower recursion

- Generate next branching "time" $t$ (after $t_{\text {previous) }}$ with probability

$$
\mathcal{P}\left(t, t_{\text {previous }}\right)=f(t) \exp \left(-\int_{t_{\text {previous }}}^{t} f\left(t^{\prime}\right) \mathrm{d} t^{\prime}\right)^{\prime}
$$

- Analytically:

$$
t=F^{-1}\left[F\left(t_{\text {previous }}\right)+\log \left(\#_{\text {random }}\right)\right] \text { with } F(t)=\int_{t_{0}}^{t} \mathrm{~d} t^{\prime} f\left(t^{\prime}\right)
$$

- If integral/its inverse are not known: "Veto algorithm" = extension of hit-or-miss
- Overestimate $g(t) \geq f(t)$ with known integral $G(t)$

$$
\rightarrow t=G^{-1}\left[G\left(t_{\text {previous }}\right)+\log \left(\#_{\text {random }}\right)\right]
$$

- Accept $t$ with probability $\frac{f(t)}{g(t)}$ using hit-or-miss


## Summary of main parton shower ingredients

- "Sudakov form factor" $\equiv$ Survival probability of parton ensemble between two scales:

$$
\mathcal{N}(t) \sim \exp \left(-\int_{0}^{t} \mathrm{~d} t^{\prime} f\left(t^{\prime}\right)\right) \rightarrow \Delta\left(t^{\prime}, t^{\prime \prime}\right)=\prod_{\{i j\}} \exp \left(-\int_{t^{\prime}}^{t^{\prime \prime}} \mathrm{d} t \mathcal{D}_{i j}^{(\mathrm{PS})}\right)
$$

- Evolution variable $t$ : not time, but scale of collinearity from hard to soft
$t \sim 2 p_{i} p_{j} \sim$ e.g. angle $\theta$, virtuality $Q^{2}$, relative transverse momentum $k_{\perp}^{2}, \ldots$
- Starting scale $\mu_{Q}^{2}$ (time $t=0$ in radioactive decay) defined by hard scattering
- Cutoff scale related to hadronisation scale $t_{0} \sim \mu_{\text {had }}^{2}$
- Other variables $(z, \phi)$ generated directly according to $\mathrm{d} \sigma_{i j}^{(\mathrm{PS})}(t, z, \phi)$


## $\Rightarrow$ Differential cross section (up to first emission)

$$
\mathrm{d} \sigma=\mathrm{d} \Phi_{B} \mathcal{B}
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$$
\mathrm{d} \sigma^{(\mathrm{B})}=\mathrm{d} \Phi_{B} \mathcal{B}[\underbrace{\Delta^{(\mathrm{PS})}\left(t_{0}, \mu_{Q}^{2}\right)}_{\text {unresolved }}+\underbrace{\sum_{\{i j\}} \int_{t_{0}}^{\mu_{Q}^{2}} \mathrm{~d} t \underbrace{\mathrm{~d} \sigma_{i j}^{(\mathrm{PS})}}_{i j} \Delta^{(\mathrm{PS} t}\left(t, \mu_{Q}^{2}\right)}_{\text {resolved }}]
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$$

## NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive $W$ production)
- Objectives:
- avoid double counting in real emission
- preserve inclusive NLO accuracy


## ME+PSRLO merging

- Multiple LO+PS simulations for processes of different jet multi (e.g. $W, W j, W j j, \ldots$ )
- Objectives:
- combine into one inclusive sample by making them exclusive
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## Combination: ME+PSRNLO

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## Reminder + Notation: Subtraction method

- Contributions to NLO cross section: $\mathcal{B o r n}, \mathcal{V}$ irtual and $\mathcal{R e a l}$ emission
- $\mathcal{V}$ and $\mathcal{R}$ divergent in separate phase space integrations
$\Rightarrow$ Subtraction terms $\mathcal{D}$ and their integrated form $\mathcal{I}$ for NLO cross section:

$$
\mathrm{d} \sigma^{(\mathrm{NLO})}=\mathrm{d} \Phi_{B}\left[\mathcal{B}+\tilde{\mathcal{V}}+\sum_{\{i j\}} \mathcal{I}_{(i j)}^{(\mathrm{S})}\right]+\mathrm{d} \Phi_{R}\left[\mathcal{R}-\sum_{\{i j\}} \mathcal{D}_{i j}^{(\mathrm{S})}\right]
$$

## Idea of NLO+PS matching

- Applying PS resummation to LO event was "simple"
- Apply the same separately for $\mathcal{B}$ and $\mathcal{V}$ and $\mathcal{R}$ at NLO?
$\Rightarrow$ double counting
- Instead: additional subtraction terms $\mathcal{D}_{i j}^{(\mathrm{A})}$

$$
\begin{aligned}
\mathrm{d} \sigma^{(\mathrm{NLO} \text { sub })} & =\mathrm{d} \Phi_{B} \overline{\mathcal{B}}^{(\mathrm{A})}+\mathrm{d} \Phi_{R}\left[\mathcal{R}-\sum_{\{i j\}} \mathcal{D}_{i j}^{(\mathrm{A})}\right] \\
\text { with } \overline{\mathcal{B}}^{(\mathrm{A})} & =\mathcal{B}+\tilde{\mathcal{V}}+\sum_{\{i j\}} \mathcal{I}_{(i j)}^{(\mathrm{S})}+\sum_{\{i j\}} \int \mathrm{d} t\left[\mathcal{D}_{i j}^{(\mathrm{A})}-\mathcal{D}_{i j}^{(\mathrm{S})}\right]
\end{aligned}
$$

## Master formula for NLO+PS up to first emission

$$
\begin{aligned}
\mathrm{d} \sigma^{(\mathrm{NLO}+\mathrm{PS})}= & \mathrm{d} \Phi_{B} \overline{\mathcal{B}}^{(\mathrm{A})}[\underbrace{\Delta^{(\mathrm{A})}\left(t_{0}, \mu_{Q}^{2}\right)}_{\text {unresolved }}+\underbrace{\sum_{\{i j\}} \int_{t_{0}}^{\mu_{Q}^{2}} \mathrm{~d} t \frac{\mathcal{D}_{i j}^{(\mathrm{A})}}{\mathcal{B}} \Delta^{(\mathrm{A})}\left(t, \mu_{Q}^{2}\right)}_{\text {resolved, singular }}] \\
& +\mathrm{d} \Phi_{R} \underbrace{\left[\mathcal{R}-\sum_{\{i j\}} \mathcal{D}_{i j}^{(\mathrm{A})}\right]}_{\text {resolved, non-singular } \equiv \mathcal{H}(\mathrm{A})}
\end{aligned}
$$

- To $\mathcal{O}\left(\alpha_{s}\right)$ this reproduces $\mathrm{d} \sigma^{(\mathrm{NLO})}$
- Event generation: $\overline{\mathcal{B}}^{(\mathrm{A})}$ or $\mathcal{H}^{(\mathrm{A})}$ seed event according to their XS
- First line ("S-event"): from one-step PS with $\Delta^{(A)}$
$\Rightarrow$ emission (resolved, singular) or no emission (unresolved) above $t_{0}$
- Second line ("H-event"): kept as-is $\rightarrow$ resolved, non-singular term
- Resolved cases: Subsequent emissions can be generated by ordinary PS
- Exact choice of $\mathcal{D}_{i j}^{(\mathrm{A})}$ will specify Mc@NLO vs. S-MC@NLO vs. POWHEG ...


## Mc®NLo

Frixione, Webber (2002)
$\mathcal{D}^{(\mathrm{A})}=$ PS splitting kernels

+ Shower algorithm for Born-like events easy to implement
- "Non-singular" piece $\mathcal{R}-\sum_{i j} \mathcal{D}_{i j}^{(\mathrm{A})}$ is actually singular:
- Collinear divergences subtracted by splitting kernels
- Remaining soft divergences as they appear in non-trivial processes at sub-leading $N_{c}$ $x$


## S-McraNlo

Höche, Krauss, Schönherr, FS (2011)

$$
\mathcal{D}^{(\mathrm{A})}=\text { Subtraction terms } \mathcal{D}^{(\mathrm{S})}
$$

+ "Non-singular" piece fully free of divergences
- Splitting kernels in shower algorithm become negative

Solution: Weighted $N_{C}=3$ one-step PS based on subtraction terms
$\Downarrow$
Used in Sherpa

Workaround: $\mathcal{G}$-function dampens soft limit in
non-singular piece
$\Leftrightarrow$ Loss of formal NLO accuracy
(but heuristically only small impact)

## Original Powheg

- Choose additional subtraction terms as

$$
\mathcal{D}_{i j}^{(\mathrm{A})} \rightarrow \rho_{i j} \mathcal{R} \quad \text { where } \quad \rho_{i j}=\frac{\mathcal{D}_{i j}^{(\mathrm{S})}}{\sum_{m n} \mathcal{D}_{m n}^{(\mathrm{S})}}
$$

- $\mathcal{H}$-term vanishes $\Rightarrow$ No negative weighted events
- Similar to PS with ME-correction for 1st emission (e.g. Herwig, Pythia)


## Mixed scheme

- Subtract arbitrary regular piece from $\mathcal{R}$ and generate separately as $\mathbb{H}$-events

$$
\mathcal{D}_{i j}^{(\mathrm{A})}\left(\Phi_{R}\right) \rightarrow \rho_{i j}\left(\Phi_{R}\right)\left[\mathcal{R}\left(\Phi_{R}\right)-\mathcal{R}^{r}\left(\Phi_{R}\right)\right] \quad \text { where } \quad \rho_{i j} \text { as above }
$$

- Tuning of $\mathcal{R}^{r}$ to reduce exponentiation of arbitrary terms
- Allows to generate the non-singular cases of $\mathcal{R}$ without underlying $\mathcal{B}$


## Inherent systematic uncertainties

## Perturbative uncertainties

- Unknown higher-order corrections
- Estimated by scale variations

$$
\mu_{F}=\mu_{R}=\frac{1}{2} \mu \ldots 2 \mu
$$

## Non-perturbative uncertainties

- Model uncertainties in hadronisation, hadron decays, multiple parton interactions
- Estimated by variation of parameters/models within tuned ranges


## Exponentiation uncertainties

- Arbitrariness of $\mathcal{D}^{(\mathrm{A})}$ and thus of the exponent in $\Delta^{(\mathrm{A})}$
- Estimated by:
- Variations of $\mu_{Q}^{2}$ in Mc@NLO
- (Variation of $\mathcal{R}^{r}$ in POWHEG)
- Reduced by merging with NLO for higher parton multiplicities $\rightsquigarrow$ later



## For demonstration purposes

- Strong sensitivity to exponentation especially at large $p_{\perp}^{\mathrm{h}}$
- POWHEG and completely unrestricted MC@NLO similar
- Decrease exponentiation of non-singular pieces using (unphysical) dipole $\alpha$ : $\alpha_{\text {cut }} \lesssim 0.01$ recovers NLO behaviour

Proper physical assessment of variation:
Dipole restriction at (and variation of) resummation scale $\mu_{Q}$

## Comparison to ATLAS data



AtLAS measurement (arXiv:1201.1276)
SHERPA+BLACKHAT NLO+PS predictions (arXiv:1201.5882)

## NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive $W$ production)
- Objectives:
- avoid double counting in real emission
- preserve inclusive NLO accuracy


## ME+PSQLO merging

- Multiple LO+PS simulations for processes of different jet multi (e.g. $W, W j, W j j, \ldots$ )
- Objectives:
- combine into one inclusive sample by making them
exclusive
- preserve resummation accuracy


Combination: ME+PSRNLO

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## Main idea

Phase space slicing for QCD radiation in shower evolution

- Hard emissions $Q_{i j}(z, t)>Q_{\text {cut }}$
- Events rejected
- Compensated by events starting from higher-order ME regularised by $Q_{\text {cut }}$
$\Rightarrow$ Splitting kernels replaced by exact real-emission matrix elements

$$
\mathcal{D}_{i j}^{(\mathrm{PS})} \quad \rightarrow \quad \mathcal{R}_{i j}
$$

(But Sudakov form factors $\Delta^{(P S)}$ remain unchanged)

- Soft/collinear emissions $Q_{i j, k}(z, t)<Q_{\text {cut }}$
$\Rightarrow$ Retained from parton shower $\quad \mathcal{D}_{i j}^{(\mathrm{PS})}=\mathcal{B} \times\left[\frac{1}{2 p_{i} p_{j}} 8 \pi \alpha_{s} \mathcal{K}_{i j}\left(p_{i}, p_{j}\right)\right]$


## Note

Boundary determined by "jet criterion" $Q_{i j, k}$

- Has to identify soft/collinear divergences in MEs, like jet algorithm
- Otherwise arbitrary


## Parton shower on top of high-multi ME

## Translate ME event into shower language

## Why? <br> ?

- Need starting scales $t$ for PS evolution
- Have to embed existing emissions into PS evolution

Problem: ME only gives final state, no history
Solution: Backward-clustering (running the shower reversed), similar to jet algorithm:

1. Select last splitting according to shower probablities

2 Recombine partons using inverted shower kinematics $\rightarrow \mathrm{N}-1$ particles + splitting variables for one node

> Example:

3 Reweight $\alpha_{s}\left(\mu^{2}\right) \rightarrow \alpha_{s}\left(p_{\perp}^{2}\right)$
4 Repeat $1-3$ until core process $(2 \rightarrow 2)$


## Truncated shower

- Shower each (external and intermediate!) line between determined scales
- "Boundary" scales: resummation scale $\mu_{Q}^{2}$ and shower cut-off $t_{0}$


## Cross section up to first emission in ME+PS

$$
\begin{aligned}
\mathrm{d} \sigma= & \mathrm{d} \Phi_{B} \mathcal{B}[\underbrace{\Delta^{(\mathrm{PS})}\left(t_{0}, \mu_{Q}^{2}\right)}_{\text {unresolved }}+\sum_{\{i j\}} \int_{t_{0}}^{\mu_{Q}^{2}} \mathrm{~d} t \Delta^{(\mathrm{PS})}\left(t, \mu^{2}\right) \\
& \times(\underbrace{\frac{\mathcal{D}_{i j}^{(\mathrm{PS})}}{\mathcal{B}} \Theta\left(Q_{\mathrm{cut}}-Q_{i j}\right)}_{\text {resolved, PS domain }}+\underbrace{\frac{\mathcal{R}_{i j}}{\mathcal{B}} \Theta\left(Q_{i j}-Q_{\mathrm{cut}}\right)}_{\text {resolved, ME domain }})]
\end{aligned}
$$

## Features

- LO weight B for Born-like event
- Unitarity slightly violated due to mismatch of $\Delta^{(\mathrm{PS})}$ and $\mathcal{R} / \mathcal{B}$ $[\ldots] \approx 1 \Rightarrow$ LO cross section only approximately preserved
- Unresolved emissions as in parton shower approach
- Resolved emissions now sliced into PS and ME domain
- Only for one emission here, but possible up to high number of emissions


## Features and shortcomings by example

## Example

Diphoton production at Tevatron

- Measured by CDF Phys.Rev.Lett. 110 (2013) 101801
- Isolated hard photons
- Azimuthal angle between the diphoton pair

ME+PS simulation using SHERPA vs. (N)NLO

## Conclusions

Shapes described very well even for this non-trivial process/observable for both:

- Hard region, e.g. $\Delta \Phi_{\gamma \gamma} \rightarrow 0$
- Soft region, e.g. $\Delta \Phi_{\gamma \gamma} \rightarrow \pi$

Scale variations high $\Rightarrow$ NLO needed



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## Concepts continued from ME+PS merging at LO

- For each event select jet multiplicity $k$ according to its inclusive NLO cross section
- Reconstruct branching history and nodal scales $t_{0} \ldots t_{k}$
- Truncated vetoed parton shower, but with peculiarities (cf. below)


## Differences for NLO merging

- For each event select type ( $\mathbb{S}$ or $\mathbb{H}$ ) according to absolute XS
$\Rightarrow$ Shower then runs differently
- S event:

1 Generate MC@NLO emission at $t_{k+1}$
2 Truncated "NLO-vetoed" shower between $t_{0}$ and $t_{k}$ : First hard emission is only ignored, no event veto

3 Continue with vetoed parton shower

- $\mathbb{H}$ event:
(Truncated) vetoed parton shower as in tree-level ME+PS

ME+PS@NLO prediction for combining NLO+PS samples of multiplicities $n$ and $n+1$

$$
\begin{aligned}
& \mathrm{d} \sigma=\mathrm{d} \Phi_{n} \overline{\mathrm{~B}}_{n}^{(\mathrm{A})}\left[\Delta_{n}^{(\mathrm{A})}\left(t_{c}, \mu_{Q}^{2}\right)+\int_{t_{c}}^{\mu_{Q}^{2}} \mathrm{~d} \Phi_{1} \frac{\mathrm{D}_{n}^{(\mathrm{A})}}{\mathrm{B}_{n}} \Delta_{n}^{(\mathrm{A})}\left(t_{n+1}, \mu_{Q}^{2}\right) \Theta\left(Q_{\mathrm{cut}}-Q_{n+1}\right)\right] \\
& +\mathrm{d} \Phi_{n+1} \mathrm{H}_{n}^{(\mathrm{A})} \Delta_{n}^{(\mathrm{PS})}\left(t_{n+1}, \mu_{Q}^{2}\right) \Theta\left(Q_{\mathrm{cut}}-Q_{n+1}\right) \\
& +\mathrm{d} \Phi_{n+1} \overline{\mathrm{~B}}_{n+1}^{(\mathrm{A})} \underbrace{\left(1+\frac{\mathrm{B}_{n+1}}{\overline{\mathrm{~B}}_{n+1}^{(\mathrm{A})}} \int_{t_{n+1}}^{\mu_{Q}^{2}} \mathrm{~d} \Phi_{1} \mathrm{~K}_{n}\right)} \Delta_{n}^{(\mathrm{PS})}\left(t_{n+1}, \mu_{Q}^{2}\right) \Theta\left(Q_{n+1}-Q_{\mathrm{cut}}\right) \\
& \times\left[\Delta_{n+1}^{(\mathrm{A})}\left(t_{c}, t_{n+1}\right)+\int_{t_{c}}^{\mathrm{MC}_{n+1}} \mathrm{~d} \Phi_{1} \frac{\mathrm{D}_{n+1}^{(\mathrm{A})}}{\mathrm{B}_{n+1}} \Delta_{n+1}^{(\mathrm{A})}\left(t_{n+2}, t_{n+1}\right)\right] \\
& +\mathrm{d} \Phi_{n+2} \\
& \mathrm{H}_{n+1}^{(\mathrm{A})} \Delta_{n+1}^{(\mathrm{PS})}\left(t_{n+2}, t_{n+1}\right) \Delta_{n}^{(\mathrm{PS})}\left(t_{n+1}, \mu_{Q}^{2}\right) \Theta\left(Q_{n+1}-Q_{\mathrm{cut}}\right)+\ldots
\end{aligned}
$$




- Significant reduction of ME+PS@NLO scale uncertainties in perturbative region
- Improved agreement with experimental data

Results for $e^{+} e^{-}$
$\rightarrow$ hadrons: Thrust event shape




- Comparison to Atlas measurement Phys.Rev. D85 (2012), 092002
- Significant reduction of ME+Ps@NLO scale uncertainties in "NLO" multiplicities
- Improved agreement with data



## Results for $W$ + jets: $H_{T}$



$H_{T}$ and related observables are sensitive to many jet multiplicities simultaneously

- Need ME+Ps@Nlo for precise description
- High $H_{T}$ region affected by higher multiplicities $\Rightarrow$ Larger scale uncertainty

Results for $W$ + jets: Angular correlations
$\Delta R$ Distance of Leading Jets


Azimuthal Distance of Leading Jets


- Pure Mc@Nlo simulation misses correlations between the two leading jets


## NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive $W$ production)
- Objectives:

- avoid double counting in real emission
- preserve inclusive NLO accuracy



## Combination: ME+PSRANLO

- Multiple NLO+PS simulations for processes of different jet multiplicity e.g. $W, W j, W j j, \ldots$
- Objectives:
- combine into one inclusive sample
- preserve NLO accuracy for jet observables


## Summary

- I'm most certainly out of time by now.


## Outlook

- Nachsitzung

