

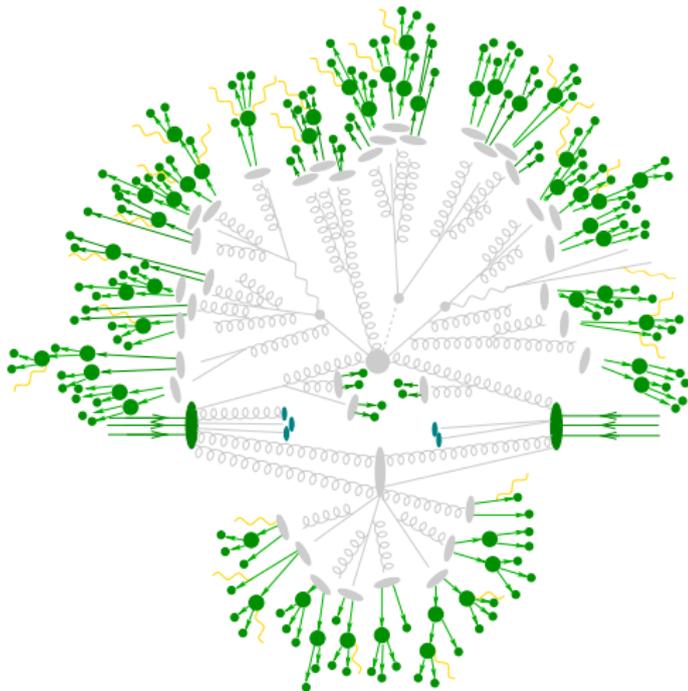


# Modern event generation for the LHC

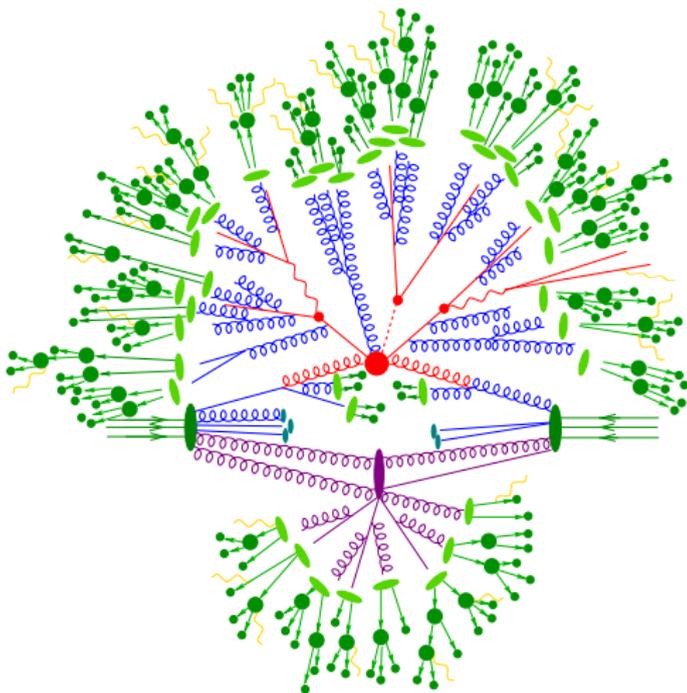
## Features and caveats of the Sherpa event generator

Frank Siegert

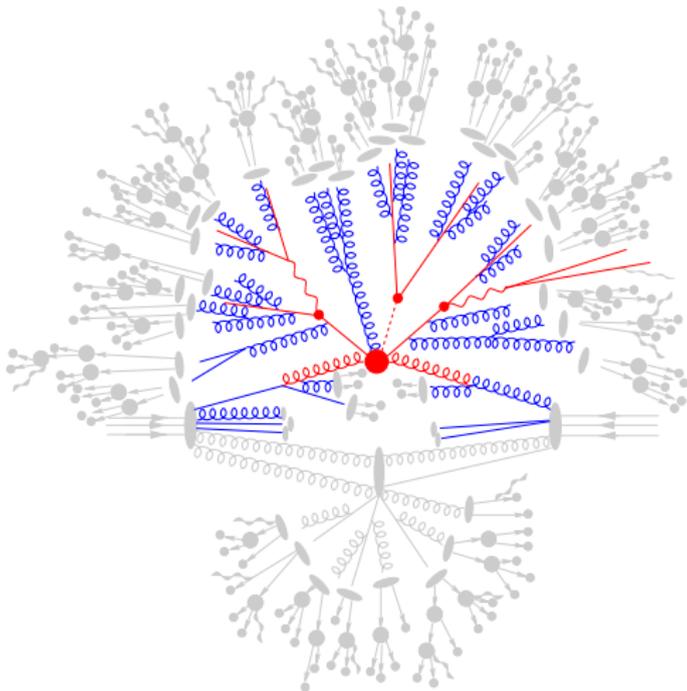
Cavendish HEP Seminar, University of Cambridge, October 2015



- We want:  
Simulation of  $pp \rightarrow$  full hadronised final state
- MC event representation (e.g.  $pp \rightarrow t\bar{t}H$  event)
- We know from first principles:
  - Hard scattering at fixed order in perturbation theory (**Matrix Element**)
  - Approximate resummation of QCD corrections to all orders (**Parton Shower**)
- Missing bits:  
Hadronisation/Underlying event (ignored today)



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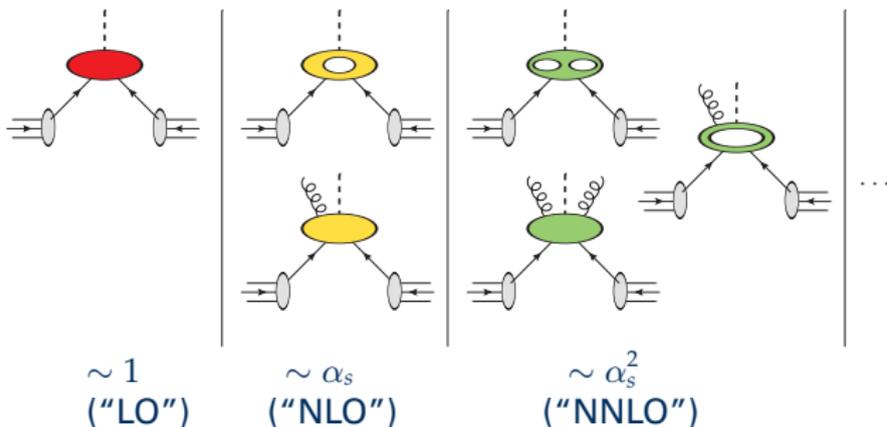


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## Outline

- Introduction
  - The parton shower approximation
  - Correcting that approximation as far as possible:
    - NLO+PS matching (2002)
    - Tree-level ME+PS merging (2001)
    - ME+PS merging at NLO (2012)
  - Practical considerations
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- Cannot solve QCD and calculate  $pp \rightarrow X$  exactly
- But can calculate parts of the perturbative series in  $\alpha_s$ :



- Most precise calculations include  $\mathcal{O}(\alpha_s^2)$  for some processes
- $\alpha_s^2 \approx 1\% \Rightarrow$  high enough precision, right?
- Why is that not always true?

- Predictions for **inclusive** observables calculable at fixed-order ( $\rightsquigarrow$  KLN theorem for cancellation of infrared divergences)
- But if **not inclusive**  $\rightarrow$  finite remainders of infrared divergences:

logarithms of  $\frac{\mu_{\text{cut}}^2}{\mu_{\text{hard}}^2}$  with each  $\mathcal{O}(\alpha_s)$

can become large and spoil convergence of perturbative series

Examples:

- Study certain regions of phase space, like  $p_{\perp}^Z \approx 0$  @ DY
- Making predictions for hadron-level final states: confinement at  $\mu_{\text{had}} \approx 1$  GeV

$\Rightarrow$  Need to resum the series to all orders

- Problem: We are not smart enough for that.
- Workaround: **Resum only the logarithmically enhanced terms in the series**

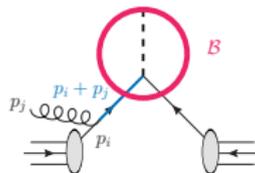
$\rightarrow$  **Parton Showers!**

# Construction of a parton shower

## Universal structure at all orders

- Factorisation of QCD real emission for collinear partons ( $i, j$ ):

$$\mathcal{R} \rightarrow \mathcal{D}_{ij}^{(\text{PS})} \equiv \mathcal{B} \times \left[ 8\pi\alpha_s \frac{1}{2p_i p_j} \mathcal{K}_{ij}(p_i, p_j) \right]$$



- Factorisation of phase space element

$$d\Phi_{\mathcal{R}} \rightarrow d\Phi_{\mathcal{B}} d\Phi_1 = d\Phi_{\mathcal{B}} dt \frac{1}{16\pi^2} dz \frac{d\phi}{2\pi}$$

with evolution variable  $t \sim 2p_i p_j \sim \theta_{ij}, k_{\perp}^{ij}, Q_{ij}$

⇒ Differential branching probability:  $d\sigma_{ij}^{(\text{PS})} \sim dt \frac{\mathcal{D}_{ij}^{(\text{PS})}}{\mathcal{B}} \sim \frac{dt}{t} \frac{\alpha_s}{2\pi} \mathcal{K}_{ij}$

- $d\sigma_{ij}^{(\text{PS})}$  is universal and appears for each emission
- How do we get the resummed branching probability according to multiple such emissions?

→ Analogy to evolution of ensemble of radioactive nuclei:

Survival probability at time  $t_1$  depends on decay/survival at times  $t < t_1$

## Radioactive decay

- Constant differential decay probability

$$f(t) = \text{const} \equiv \lambda$$

- Survival probability  $\mathcal{N}(t)$

$$-\frac{d\mathcal{N}}{dt} = \lambda \mathcal{N}(t)$$

$$\Rightarrow \mathcal{N}(t) \sim \exp(-\lambda t)$$

- Resummed decay probability  $\mathcal{P}(t)$

$$\mathcal{P}(t) = f(t) \mathcal{N}(t) \sim \lambda \exp(-\lambda t)$$

## Parton shower branching

- Differential branching probability

$$f(t) \equiv \frac{\mathcal{D}_{ij}^{(\text{PS})}}{\mathcal{B}}$$

- Survival probability  $\mathcal{N}(t)$

$$-\frac{d\mathcal{N}}{dt} = f(t) \mathcal{N}(t)$$

$$\Rightarrow \mathcal{N}(t) \sim \exp\left(-\int_0^t dt' f(t')\right)$$

- Resummed branching probability  $\mathcal{P}(t)$

$$\mathcal{P}(t) = f(t) \mathcal{N}(t) \sim f(t) \exp\left(-\int_0^t dt' f(t')\right)$$

## Summary of main parton shower ingredients

- “Sudakov form factor”  $\equiv$  Survival probability of parton ensemble:

$$\mathcal{N}(t) \sim \exp\left(-\int_0^t dt' f(t')\right) \quad \rightarrow \quad \Delta(t', t'') = \prod_{\{ij\}} \exp\left(-\int_{t'}^{t''} dt \frac{\mathcal{D}_{ij}^{(\text{PS})}}{\mathcal{B}}\right)$$

- Evolution variable  $t$ : not time, but scale of collinearity from hard to soft  
 $t \sim 2p_i p_j \sim$  e.g. angle  $\theta$ , virtuality  $Q^2$ , relative transverse momentum  $k_{\perp}^2, \dots$
- Starting scale  $\mu_Q^2$  (time  $t = 0$  in radioactive decay) defined by hard scattering
- Cutoff scale related to hadronisation scale  $t_0 \sim \mu_{\text{had}}^2$
- Other variables  $(z, \phi)$  generated directly according to  $d\sigma_{ij}^{(\text{PS})}(t, z, \phi)$

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$\Rightarrow$  **Differential cross section** (up to first emission)

$$d\sigma^{(\text{B})} = d\Phi_B \mathcal{B}$$

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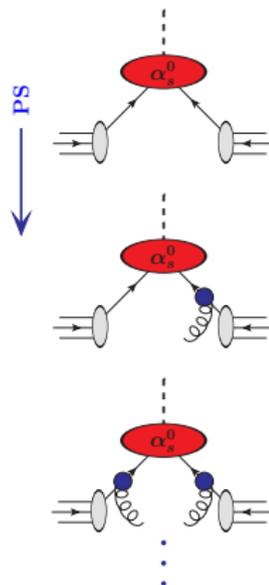
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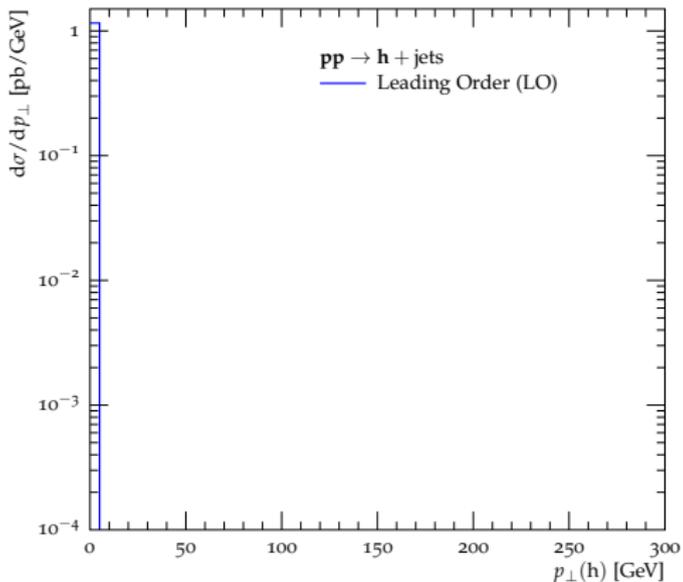
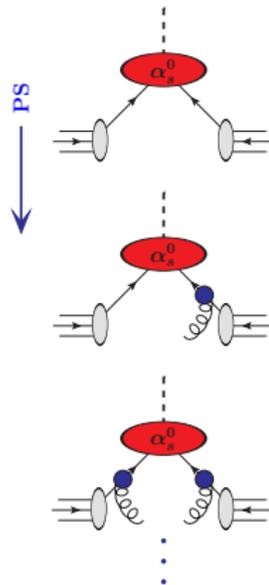
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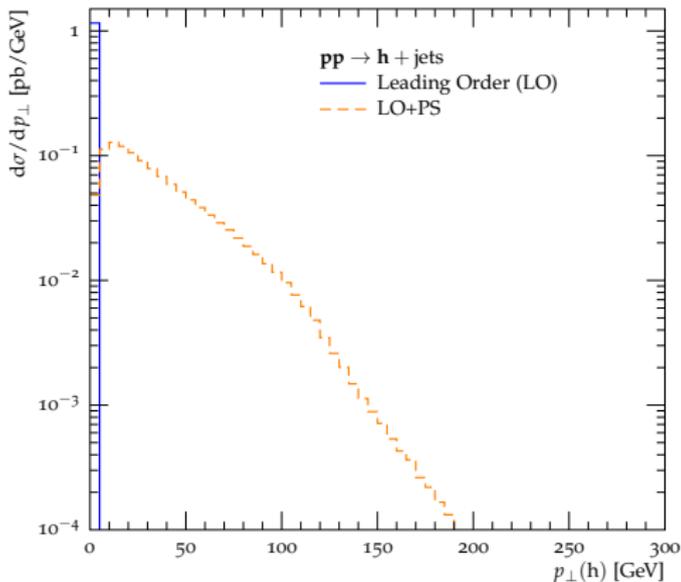
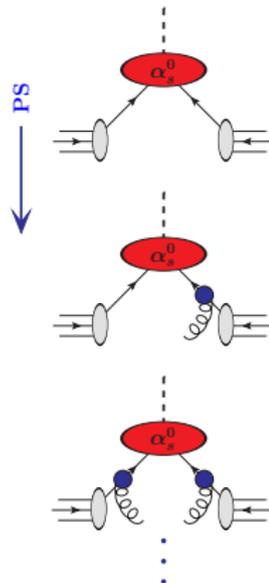
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# Parton shower improvements

## NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive  $W$  production)
- Objectives:
  - avoid double counting in real emission
  - preserve inclusive NLO accuracy



## ME+PS@LO merging

- Multiple LO+PS simulations for processes of different jet multiplicity (e.g.  $W$ ,  $W_j$ ,  $W_{jj}$ , ...)
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## Combination: ME+PS@NLO

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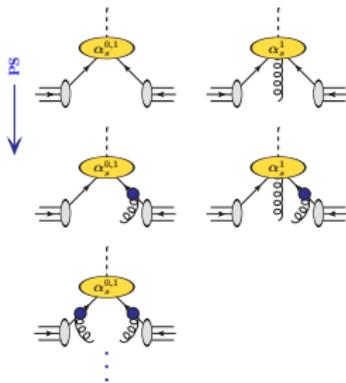
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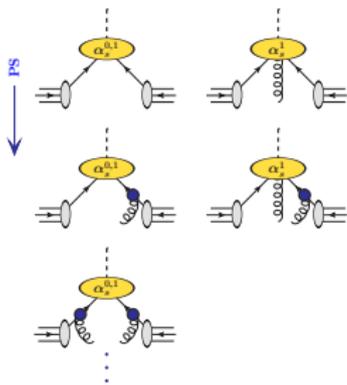
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## Basic idea



- “double-counting” between emission in real ME and parton shower for first emission
- ME is better than PS → subtract PS contribution first
- but: shower unitary → re-add “integrated” PS contribution with Born kinematics



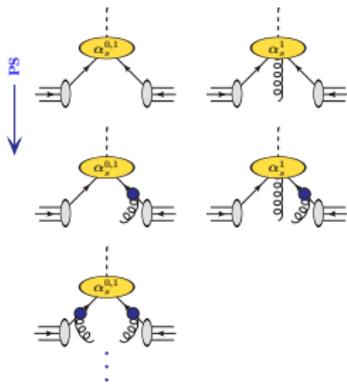
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## Subtlety: NLO already contains subtraction

$$d\sigma^{(\text{NLO})} = d\Phi_B \left[ \mathcal{B} + \tilde{\mathcal{V}} + \sum_{\{ij\}} \mathcal{I}_{(ij)}^{(S)} \right] + d\Phi_R \left[ \mathcal{R} - \sum_{\{ij\}} \mathcal{D}_{ij}^{(S)} \right]$$

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## Additional subtraction

- introduce additional (shower) subtraction terms  $\mathcal{D}_{ij}^{(A)}$

$$d\sigma^{(\text{NLO sub})} = d\Phi_B \bar{\mathcal{B}}^{(A)} + d\Phi_R \left[ \mathcal{R} - \sum_{\{ij\}} \mathcal{D}_{ij}^{(A)} \right]$$

$$\text{with } \bar{\mathcal{B}}^{(A)} = \mathcal{B} + \tilde{\mathcal{V}} + \sum_{\{ij\}} \mathcal{I}_{ij}^{(S)} + \sum_{\{ij\}} \int dt \left[ \mathcal{D}_{ij}^{(A)} - \mathcal{D}_{ij}^{(S)} \right]$$

- now apply PS resummation using  $\mathcal{D}_{ij}^{(A)}$  as splitting kernels

## Master formula for NLO+PS up to first emission

$$\begin{aligned}
 d\sigma^{(\text{NLO+PS})} = & d\Phi_B \bar{\mathcal{B}}^{(A)} \left[ \underbrace{\Delta^{(A)}(t_0, \mu_Q^2)}_{\text{unresolved}} + \underbrace{\sum_{\{ij\}} \int_{t_0}^{\mu_Q^2} dt \frac{\mathcal{D}_{ij}^{(A)}}{\mathcal{B}} \Delta^{(A)}(t, \mu_Q^2)}_{\text{resolved, singular}} \right] \\
 & + d\Phi_R \left[ \underbrace{\mathcal{R} - \sum_{\{ij\}} \mathcal{D}_{ij}^{(A)}}_{\text{resolved, non-singular} \equiv \mathcal{H}^{(A)}} \right]
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 \end{aligned}$$

- To  $\mathcal{O}(\alpha_s)$  this reproduces  $d\sigma^{(\text{NLO})}$
- Exact choice of  $\mathcal{D}_{ij}^{(A)}$  distinguishes MC@NLO vs. POWHEG vs. S-MC@NLO vs. ...
- Resolved cases: Subsequent emissions can be generated by ordinary PS

## Mc@NLO

Frixione, Webber (2002)

$\mathcal{D}^{(A)} = \mathcal{D}^{(\text{PS})} = \text{PS splitting kernels}$

- + Shower algorithm for Born-like events easy to implement
- "Non-singular" piece  $\mathcal{R} - \sum_{ij} \mathcal{D}_{ij}^{(A)}$  is actually **singular**:
  - Collinear divergences subtracted by splitting kernels ✓
  - Remaining soft divergences as they appear in non-trivial processes at sub-leading  $N_c$  ✗

**Workaround:**  $\mathcal{G}$ -function dampens soft limit in non-singular piece  
 $\Leftrightarrow$  Loss of formal NLO accuracy but heuristically shown to be negligible

## S-Mc@NLO

Höhe, Krauss, Schönherr, FS (2011)

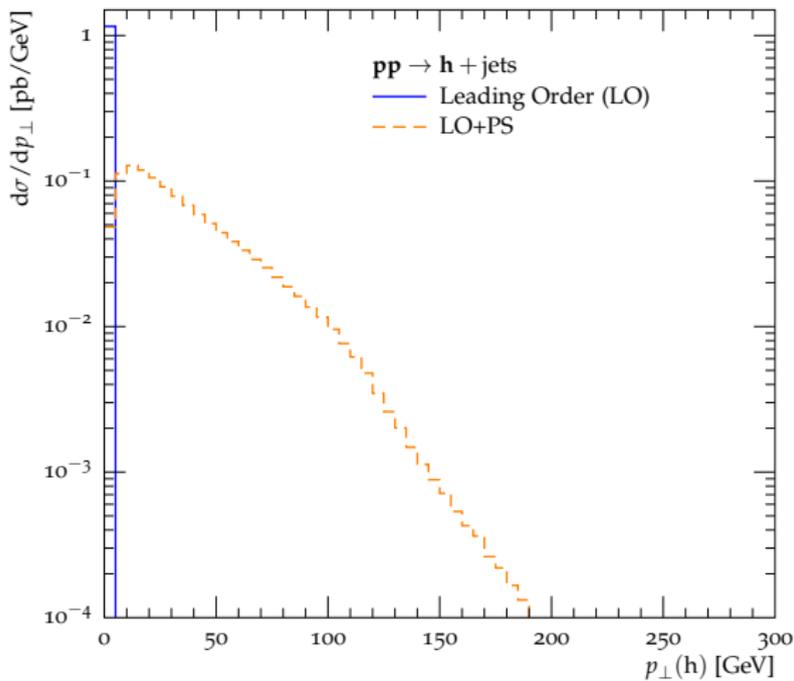
$\mathcal{D}^{(A)} = \mathcal{D}^{(S)} = \text{Subtraction terms}$

- + "Non-singular" piece fully free of divergences
- Splitting kernels in shower algorithm become **negative**

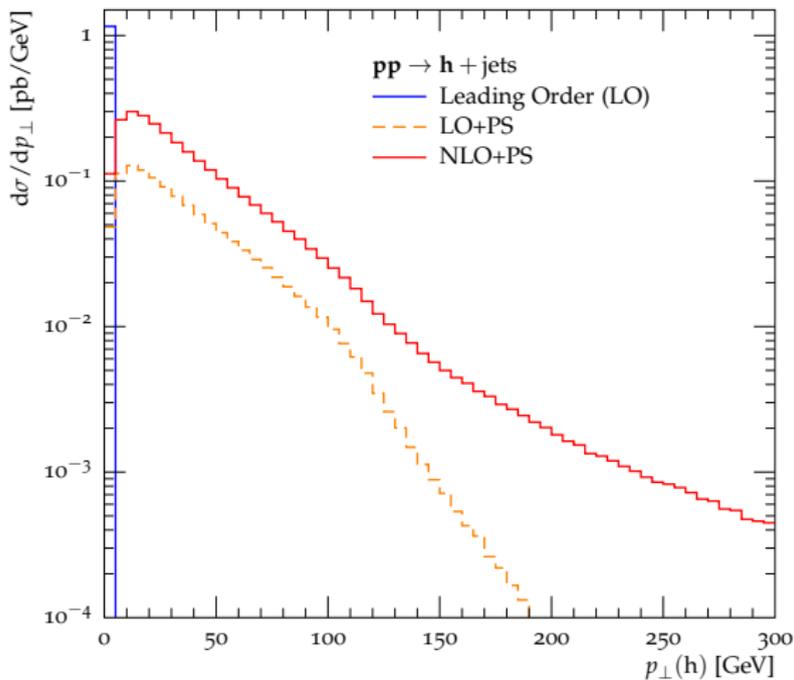
**Solution:** Weighted  $N_c = 3$  one-step PS based on subtraction terms

$\Downarrow$   
 Used in SHERPA

## Example: $pp \rightarrow h + \text{jets}$



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## NLO+PS matching

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## ME+PS@LO merging

- Multiple LO+PS simulations for processes of different jet multiplicity (e.g.  $W, W_j, W_{jj}, \dots$ )
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## Main idea

Catani, Krauss, Kuhn, Webber (2001)

Phase space slicing for QCD radiation in shower evolution

- **Soft/collinear emissions**  $Q_{ij}(z, t) < Q_{\text{cut}}$   
 $\Rightarrow$  Retained from parton shower  $\mathcal{D}_{ij}^{(\text{PS})} = \mathcal{B} \times \left[ 8\pi\alpha_s \frac{1}{2p_i p_j} \mathcal{K}_{ij}(p_i, p_j) \right]$
- **Hard emissions**  $Q_{ij}(z, t) > Q_{\text{cut}}$ 
  - Events rejected  $\rightsquigarrow$  Sudakov suppression
  - Compensated by events starting from higher-order ME regularised by  $Q_{\text{cut}}$ $\Rightarrow$  Splitting kernels replaced by exact real-emission matrix elements

$$\mathcal{D}_{ij}^{(\text{PS})} \rightarrow \mathcal{R}_{ij}$$

(But Sudakov form factors  $\Delta^{(\text{PS})}$  remain unchanged)

## Cross section up to first emission in ME+PS

$$\begin{aligned}
 d\sigma = d\Phi_B \mathcal{B} & \left[ \underbrace{\Delta^{(\text{PS})}(t_0, \mu_Q^2)}_{\text{unresolved}} + \sum_{\{ij\}} \int_{t_0}^{\mu_Q^2} dt \Delta^{(\text{PS})}(t, \mu^2) \right. \\
 & \left. \times \left( \underbrace{\frac{\mathcal{D}_{ij}^{(\text{PS})}}{\mathcal{B}} \Theta(Q_{\text{cut}} - Q_{ij})}_{\text{resolved, PS domain}} + \underbrace{\frac{\mathcal{R}_{ij}}{\mathcal{B}} \Theta(Q_{ij} - Q_{\text{cut}})}_{\text{resolved, ME domain}} \right) \right]
 \end{aligned}$$

## Translate ME event into shower language

### Embedding existing emissions into PS evolution

- Preserve resummation features (logarithmic accuracy)
- Determine starting scales  $t$  for PS evolution

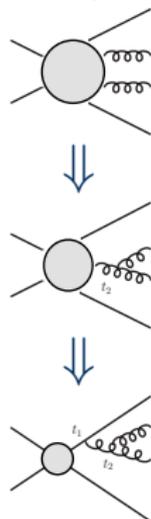
⇒ Shower picture of ME event needed!

Problem: ME only gives final state, no history

Solution: Backward-clustering (running the shower reversed), similar to jet algorithm:

- 1 Select last splitting according to shower probabilities
- 2 Recombine partons using inverted shower kinematics  
→ N-1 particles + splitting variables for one node
- 3 Reweight  $\alpha_s(\mu^2) \rightarrow \alpha_s(p_{\perp}^2)$
- 4 Repeat 1 - 3 until core process (2 → 2)

Example:



## Truncated shower

- Shower each (external and intermediate!) line between determined scales
- “Boundary” scales: resummation scale  $\mu_Q^2$  and shower cut-off  $t_0$

## Example

Diphoton production at Tevatron

- Measured by CDF [Phys.Rev.Lett. 110 \(2013\) 101801](#)
- Isolated hard photons
- Azimuthal angle between the diphoton pair

ME+PS simulation using SHERPA

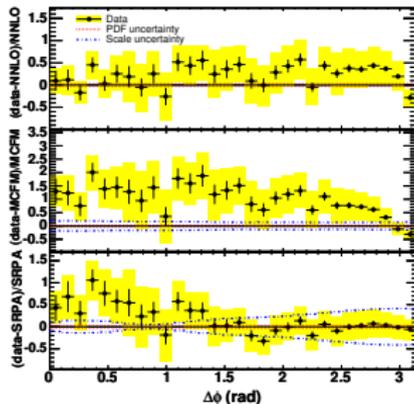
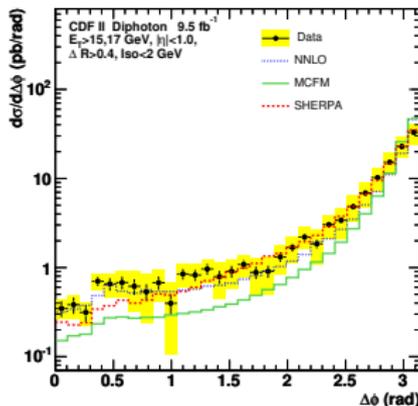
[Höche, Schumann, FS \(2009\)](#)

## Conclusions

Shapes described very well even for this non-trivial process/observable for both:

- Hard region, e.g.  $\Delta\Phi_{\gamma\gamma} \rightarrow 0$
- Soft region, e.g.  $\Delta\Phi_{\gamma\gamma} \rightarrow \pi$

Scale variations high  $\Rightarrow$  NLO needed



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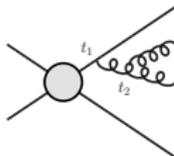
## Concepts continued from ME+PS merging at LO

- For each event select jet multiplicity  $k$  according to its inclusive NLO cross section
- Reconstruct branching history and nodal scales  $t_0 \dots t_k$
- Truncated vetoed parton shower, but with peculiarities (cf. below)

## Differences for NLO merging

- For each event select type ( $\mathbb{S}$  or  $\mathbb{H}$ ) according to absolute XS  
 $\Rightarrow$  Shower then runs differently
- $\mathbb{S}$  event:
  - 1 Generate MC@NLO emission at  $t_{k+1}$
  - 2 Truncated “NLO-vetoed” shower between  $t_0$  and  $t_k$ :  
 First hard emission is only ignored, no event veto
  - 3 Continue with vetoed parton shower
- $\mathbb{H}$  event:  
 (Truncated) vetoed parton shower as in tree-level ME+PS

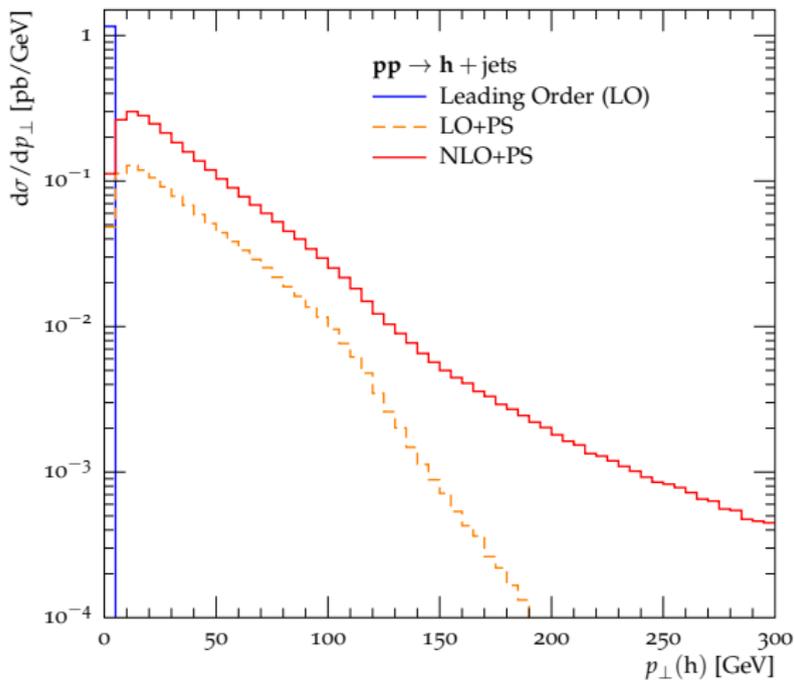
Example:  $k = 1$



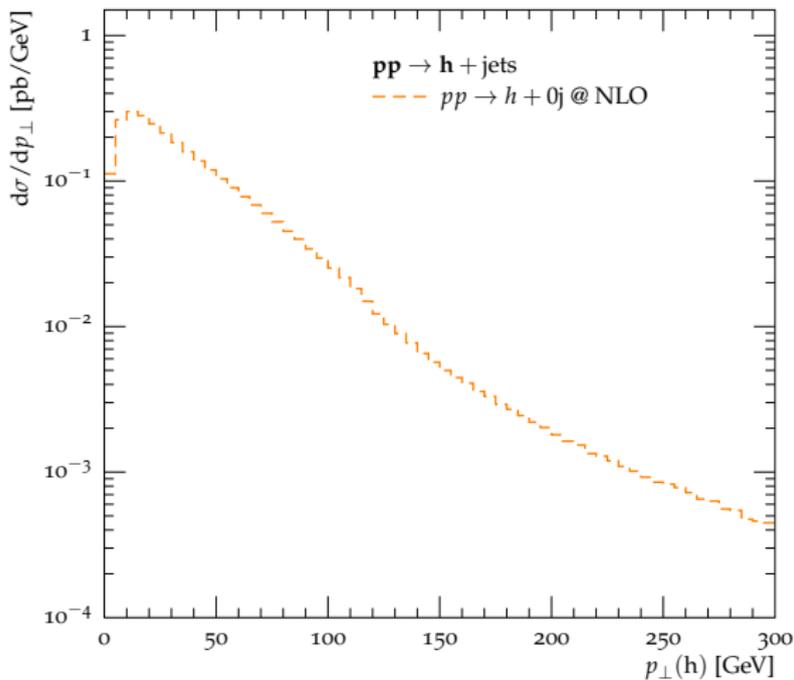
ME+PS@NLO prediction for combining NLO+PS samples of multiplicities  $n$  and  $n + 1$

$$\begin{aligned}
 d\sigma = & d\Phi_n \bar{B}_n^{(A)} \left[ \Delta_n^{(A)}(t_c, \mu_Q^2) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \frac{D_n^{(A)}}{B_n} \Delta_n^{(A)}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q_{n+1}) \right] \\
 & + d\Phi_{n+1} H_n^{(A)} \Delta_n^{(\text{PS})}(t_{n+1}, \mu_Q^2) \Theta(Q_{\text{cut}} - Q_{n+1}) \\
 & + d\Phi_{n+1} \bar{B}_{n+1}^{(A)} \underbrace{\left( 1 + \frac{B_{n+1}}{\bar{B}_{n+1}^{(A)}} \int_{t_{n+1}}^{\mu_Q^2} d\Phi_1 K_n \right)}_{\text{MC counterterm} \rightarrow \text{NLO-vetoed shower}} \Delta_n^{(\text{PS})}(t_{n+1}, \mu_Q^2) \Theta(Q_{n+1} - Q_{\text{cut}}) \\
 & \times \left[ \Delta_{n+1}^{(A)}(t_c, t_{n+1}) + \int_{t_c}^{t_{n+1}} d\Phi_1 \frac{D_{n+1}^{(A)}}{B_{n+1}} \Delta_{n+1}^{(A)}(t_{n+2}, t_{n+1}) \right] \\
 & + d\Phi_{n+2} H_{n+1}^{(A)} \Delta_{n+1}^{(\text{PS})}(t_{n+2}, t_{n+1}) \Delta_n^{(\text{PS})}(t_{n+1}, \mu_Q^2) \Theta(Q_{n+1} - Q_{\text{cut}}) + \dots
 \end{aligned}$$

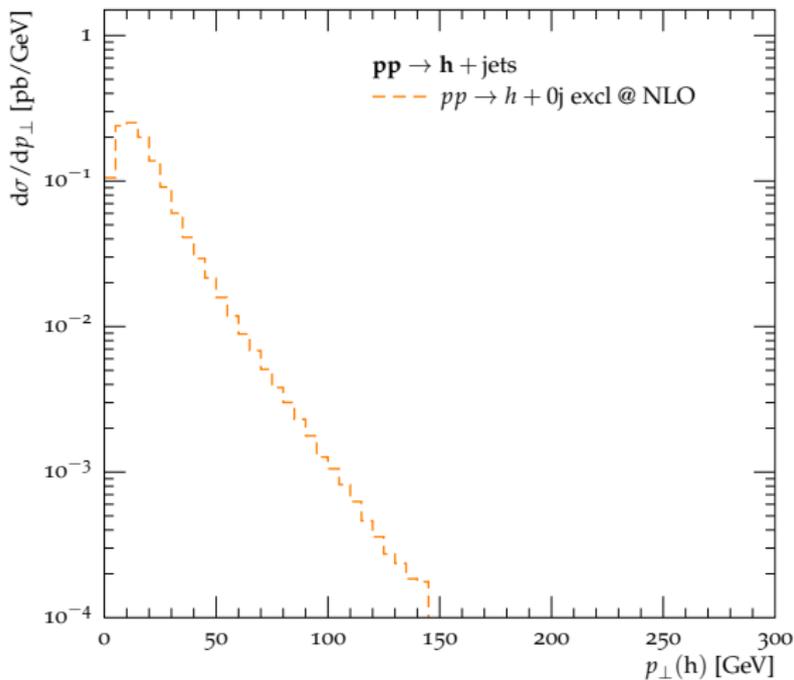
## Example: $pp \rightarrow h + \text{jets}$



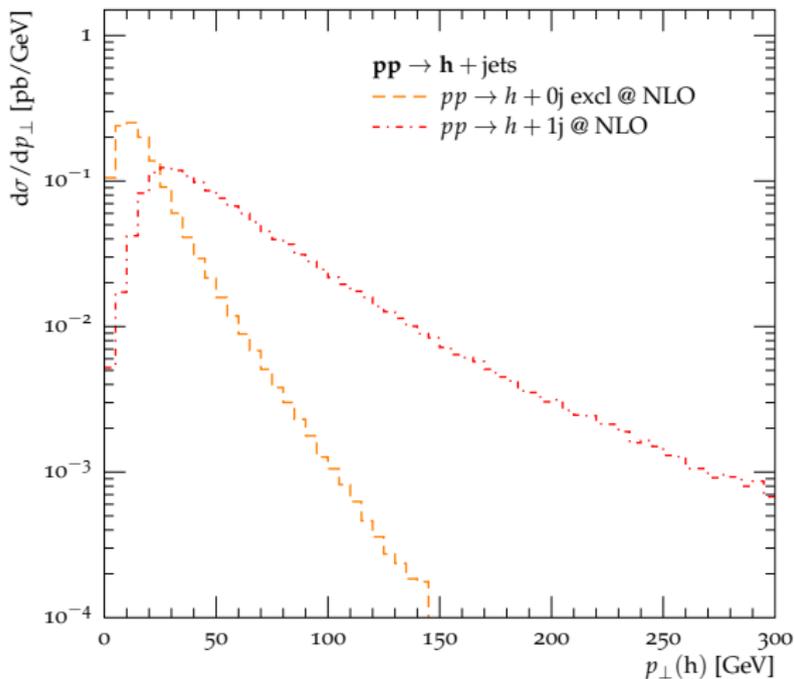
## Example: $pp \rightarrow h + \text{jets}$



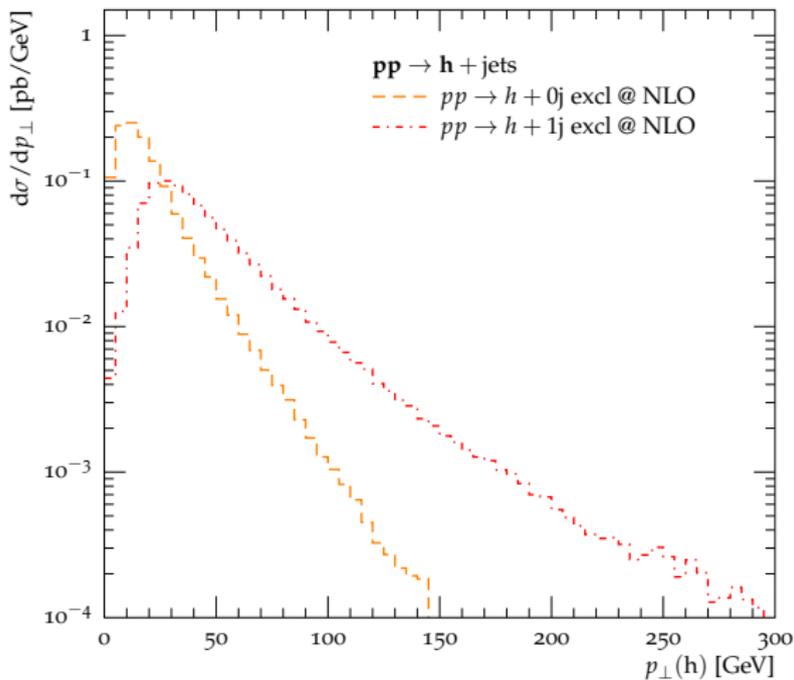
## Example: $pp \rightarrow h + \text{jets}$



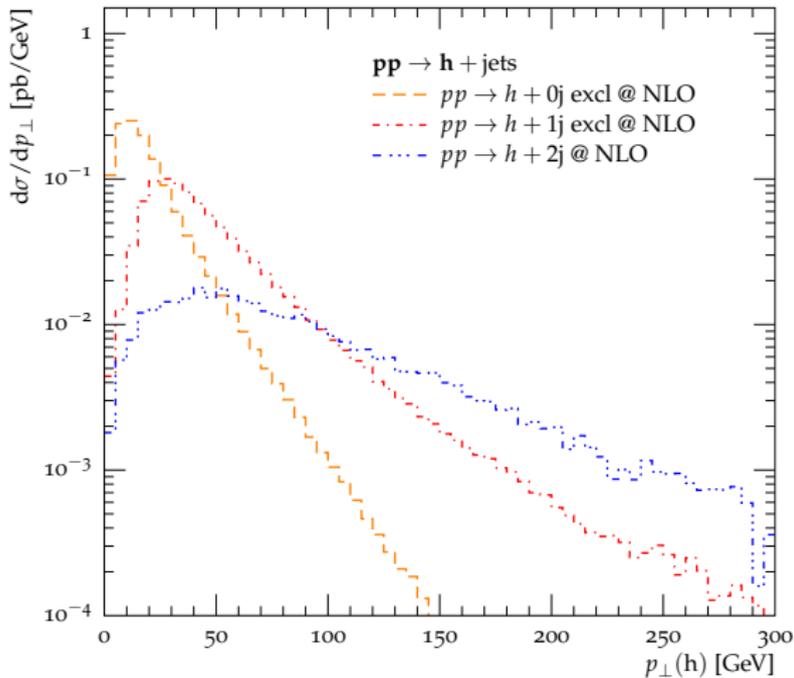
## Example: $pp \rightarrow h + \text{jets}$



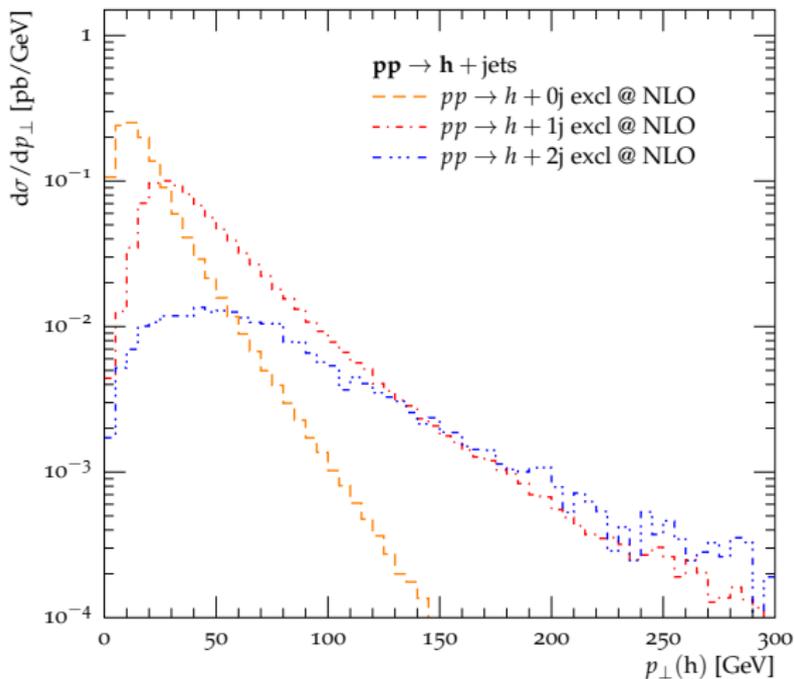
## Example: $pp \rightarrow h + \text{jets}$



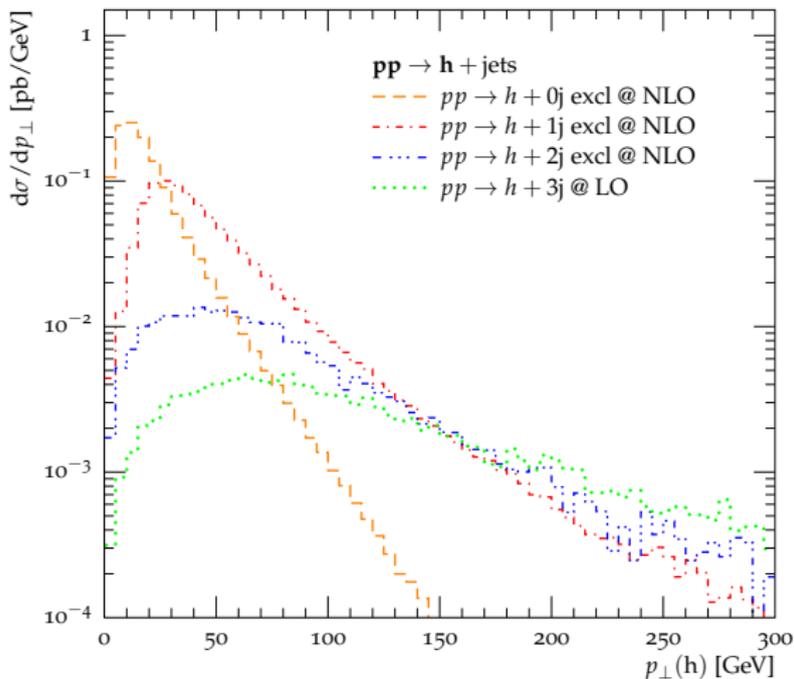
## Example: $pp \rightarrow h + \text{jets}$



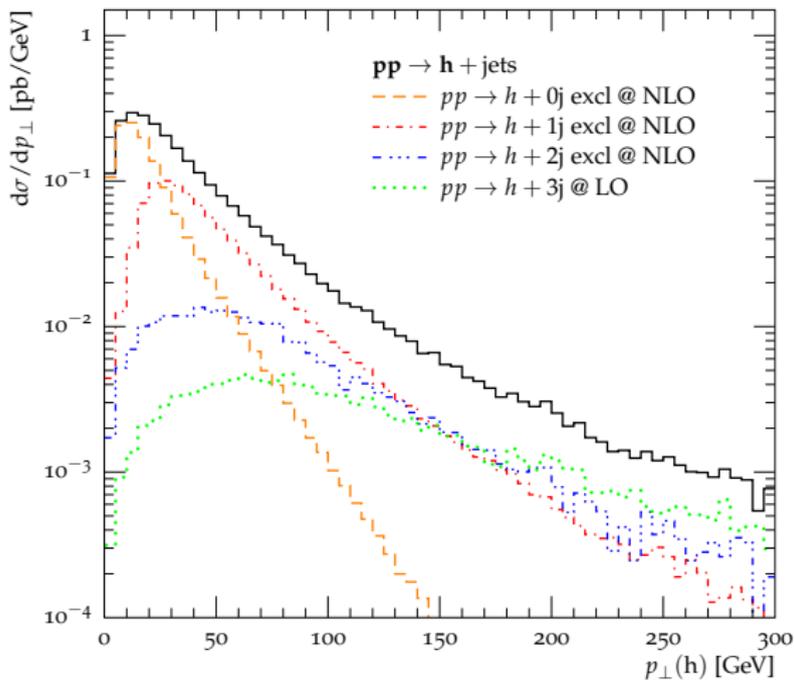
## Example: $pp \rightarrow h + \text{jets}$



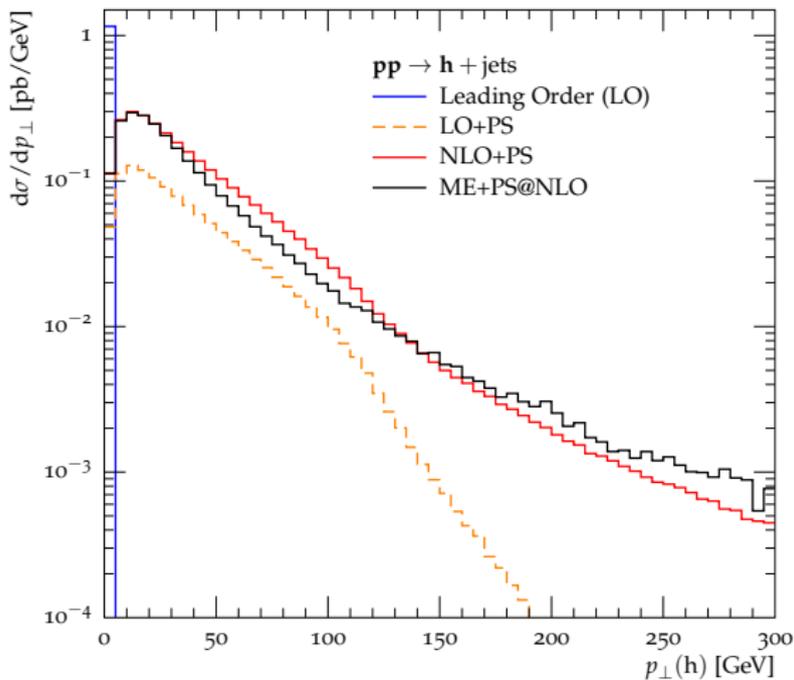
## Example: $pp \rightarrow h + \text{jets}$



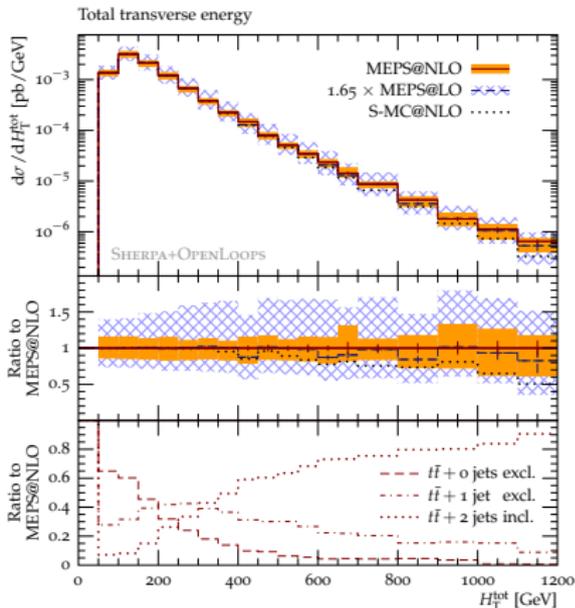
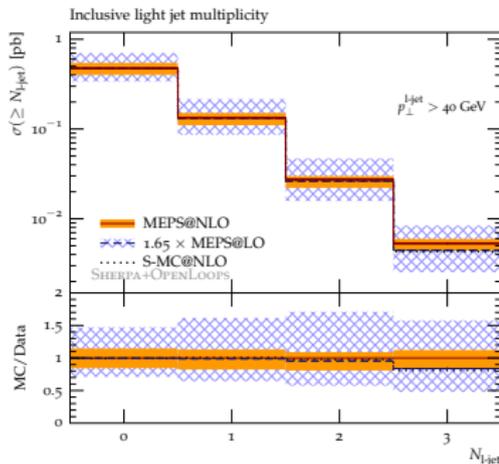
## Example: $pp \rightarrow h + \text{jets}$



## Example: $pp \rightarrow h + \text{jets}$



Höche, Krauss, Maierhöfer, Pozzorini, Schönherr, FS (2014)



- Uncertainty reduction from 79% to 19% in 2-jet bin
- Important BSM search selection: high total transverse energy  
 → major reduction of theoretical uncertainties compared to tree-level merging

# Practical considerations

## Perturbative uncertainties

- Unknown higher-order corrections
- Estimated by scale variations

$$\mu_F = \mu_R = \frac{1}{2}\mu \dots 2\mu$$

## Non-perturbative uncertainties

- Model uncertainties in hadronisation, hadron decays, multiple parton interactions, parton shower (evolution variable, kinematics reshuffling, infrared cut-offs)
- Estimated by variation of parameters/models within tuned ranges

## Matching/merging uncertainties

- Arbitrariness of  $\mathcal{D}^{(A)}$  and thus of the exponent in  $\Delta^{(A)}$ 
  - Estimated by:
    - Variations of  $\mu_Q^2$  in MC@NLO
    - (Variation of  $\mathcal{R}^f$  in POWHEG)
  - Reduced by merging with higher parton multiplicities
- Choice of merging cut

## Scale vs. core scale

- Multi-jet matrix elements embedded into parton shower evolution  
⇒ Extra emissions should be evaluated with same  $\alpha_s(p_{\perp}^{ij})$
- Remaining freedom: core process scales  $\mu_R, \mu_F$

## Global scale setting

(SCALES parameter)

- For multi-jet merged samples the METS setter has to be used to implement the above
- For simpler fixed-order or (N)LO+PS samples without merging:
  - VAR scale setter for arbitrary functions of parton level momenta
  - FASTJET scale setter to use jet momenta
  - custom definition by writing C++ code

## Core process scale

(CORE\_SCALE parameter)

- For use with the METS global scale setter
- Different options to define the core scale:
  - VAR core scale setter for arbitrary functions of parton level momenta
  - DEFAULT core scale setter for automatically taking into account type of core process (cf. next slide)

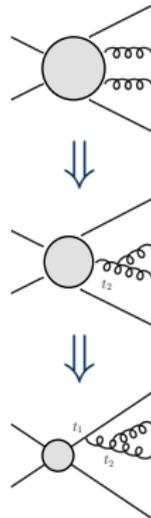
- Multi-jet merging based on core process + up to  $n$  partons
- Core process defined by user → unambiguous?  
→ two options to translate ME events into shower language:

## “Exclusive” merging

(EXCLUSIVE\_CLUSTER\_MODE=1)

- Emission history identified by QCD clustering only  
⇒ core process as defined by user
- Most straightforward way of a shower history
- If core process contains partons, e.g. in electroweak V+2-jets production: parton level cuts for “core” jets necessary
- What if the event looks more like hard QCD with softer EW attached?

Example:



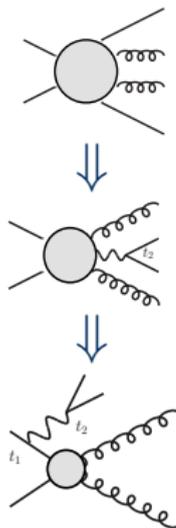
- Multi-jet merging based on core process + up to  $n$  partons
- Core process defined by user → unambiguous?  
→ two options to translate ME events into shower language:

## “Inclusive” merging

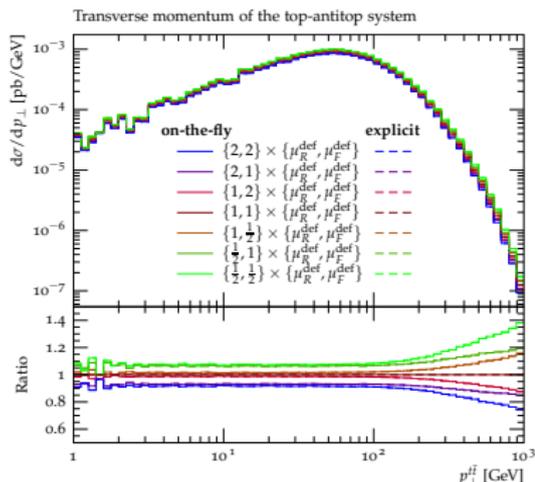
(EXCLUSIVE\_CLUSTER\_MODE=0)

- Allow EW clusterings in emission history  
⇒ can end up with different core process
- This core process is then also used to define the core scales (e.g. factorisation scale)

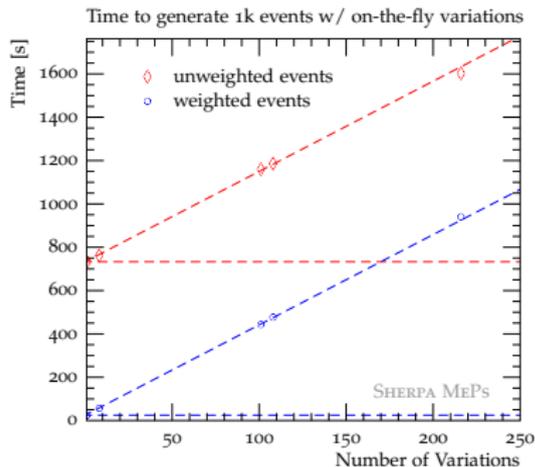
Example:



- Dedicated scale/PDF variation runs expensive, unfeasible for PDF4LHC prescription
- Instead: simultaneously keep track of variations in ME by multiple event weights
- Available since Sherpa 2.2.0 for fixed-order, S-Mc@NLO and ME+PS@LO simulations
- Upcoming in next release for ME+PS@NLO as well



- Closure compared to variation in dedicated runs for  $pp \rightarrow t\bar{t}W$  with S-Mc@NLO



- Mainly useful for expensive MEs and unweighted events

## Functional form $Q_{ij}(z, t)$

ME+PS separation determined by “jet criterion”  $Q_{ij}(z, t) > Q_{\text{cut}}$

- Has to identify soft/collinear divergences in MEs, like jet algorithm
- Otherwise arbitrary functional form

## Cut value $Q_{\text{cut}}$

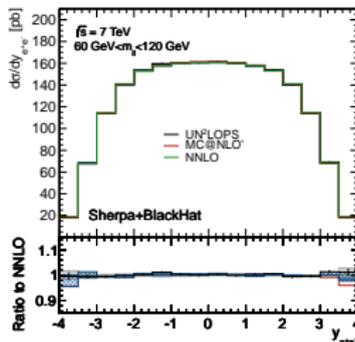
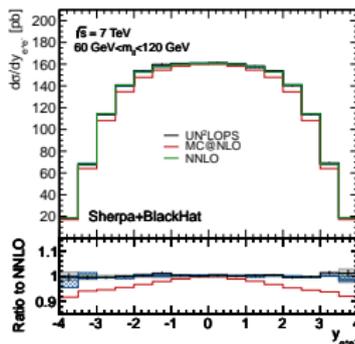
- If the merging prescription works well: ME region is supplemented consistently with resummation  
⇒ merging cut can be chosen arbitrarily low
- Disadvantages of very low merging cuts:
  - higher proportion of multi-parton MEs ⇒ CPU time increases
  - MEs less stable, integration converges more slowly⇒ Typically compromise between physics and costs: merging cut softer than typical jet criterion in analysis (e.g.  $p_{\perp} > 20 \text{ GeV}$ )
- Careful with extreme phase space regions (e.g. very forward jets)!
- Dynamical definition of merging cuts for special circumstances, e.g. in photon production to capture fragmentation component

## Consistent PDF and $\alpha_S$ usage

- Sherpa uses the same PDF and corresponding  $\alpha_S$  parametrisation everywhere: MEs, parton shower, MPI, ...
- This implies that varying the PDF also changes the parton shower and MPI behaviour
- Since recently using NNLO PDF as default
  - PDF fits sensitive to *shapes*, ME+PS merging captures N(N(...))LO *shapes*
  - Some inclusive processes benefit significantly from usage of more reliable NNLO PDF

## Tuning

- Tuning is done with a given PDF, e.g. default in Sherpa 2.2 is NNPDF 3.0 NNLO
- Should one change the full tune when using different PDFs (double counting of systematic uncertainties)? (irrelevant for on-the-fly PDF variation, since that is only available for MEs)



## Negative weights

- NLO-matched simulations  $\Rightarrow$  negative weight events from subtraction terms and  $N_C = 3$  shower
- Fraction of negative weights can vary
  - $r = \text{few \%}$  for simple processes
  - $r = 20 - 30\%$  for complex ones

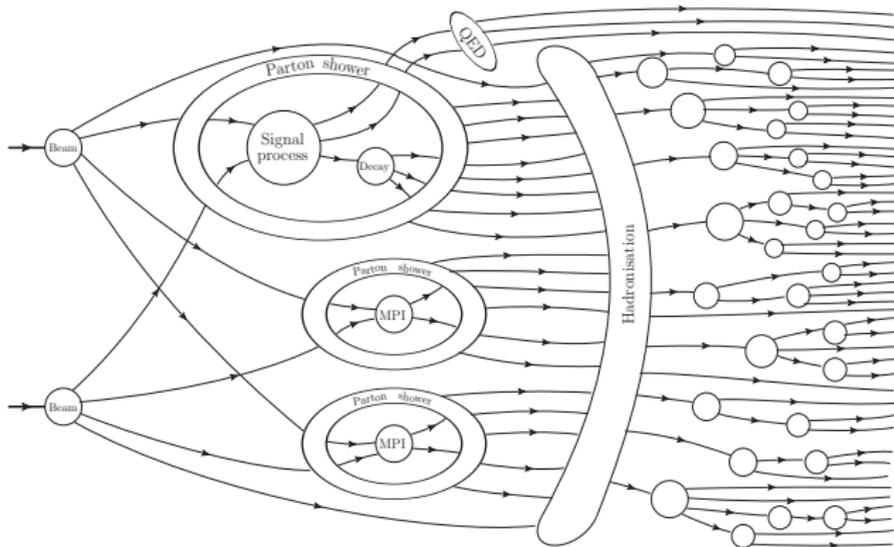
$\Rightarrow$  effective statistical uncertainty increased by factor  $\frac{1}{1-2r}$

## Weight distributions

- Unweighted events would ideally have weights reduced to  $\pm 1$
- There are a few additional weights which can not be removed by unweighting
  - "Overweight" events: phase space point yields ME value larger than the maximum found during integration
  - $N_C = 3$  shower weights
  - "local K-factor" for LO multiplicities on top of NLO

$\Rightarrow$  (steeply falling) weight distribution around  $\pm 1$

- Experiment software prefers tree-like event records with straightened mother-daughter relations
- This is not necessarily the case in Sherpa:



- Dipole-like parton showers imply there is no distinction between ISR and FSR  
⇒ Parton shower “blob” can lead to particle “loops”
- (New option in Sherpa 2.2 removes the inside of shower blobs to give straight event record)

# Conclusions

## NLO+PS matching

- Parton shower on top of NLO prediction (e.g. inclusive  $W$  production)
- Objectives: 
  - avoid double counting in real emission
  - preserve inclusive NLO accuracy



## ME+PS@LO merging

- Multiple LO+PS simulations for processes of different jet multiplicity (e.g.  $W$ ,  $W_j$ ,  $W_{jj}$ , ...)
- Objectives: 
  - combine into one inclusive sample by making them exclusive
  - preserve resummation accuracy



## Combination: ME+PS@NLO

- Multiple NLO+PS simulations for processes of different jet multiplicity e.g.  $W$ ,  $W_j$ ,  $W_{jj}$ , ...
- Objectives:  
  - combine into one inclusive sample
  - preserve NLO accuracy for jet observables

## Outlook

- Skipped today: NNLO+PS matching in Sherpa [Höhe, Li, Prestel \(2014\)](#)
- ⇒ **Perturbative** accuracy covered with new approaches in recent years
- Big effort on bringing the improvements into full **production** within experiments
  - Experimental validation
  - Feasibility for (unweighted) event generation with highest accuracies
  - User support for new practical issues
- Future focus on improvement of **resummation accuracy** in parton showers

# Backup

## Original POWHEG

- Choose additional subtraction terms as

$$\mathcal{D}_{ij}^{(A)} \rightarrow \rho_{ij} \mathcal{R} \quad \text{where} \quad \rho_{ij} = \frac{\mathcal{D}_{ij}^{(S)}}{\sum_{mn} \mathcal{D}_{mn}^{(S)}}$$

- $\mathcal{H}$ -term vanishes  $\Rightarrow$  No negative weighted events
- Similar to PS with ME-correction for 1st emission (e.g. Herwig, Pythia)

## Mixed scheme

- Subtract arbitrary regular piece from  $\mathcal{R}$  and generate separately as  $\mathbb{H}$ -events

$$\mathcal{D}_{ij}^{(A)}(\Phi_R) \rightarrow \rho_{ij}(\Phi_R) [\mathcal{R}(\Phi_R) - \mathcal{R}^r(\Phi_R)] \quad \text{where} \quad \rho_{ij} \text{ as above}$$

- Tuning of  $\mathcal{R}^r$  to reduce exponentiation of arbitrary terms
- Allows to generate the non-singular cases of  $\mathcal{R}$  without underlying  $\mathcal{B}$